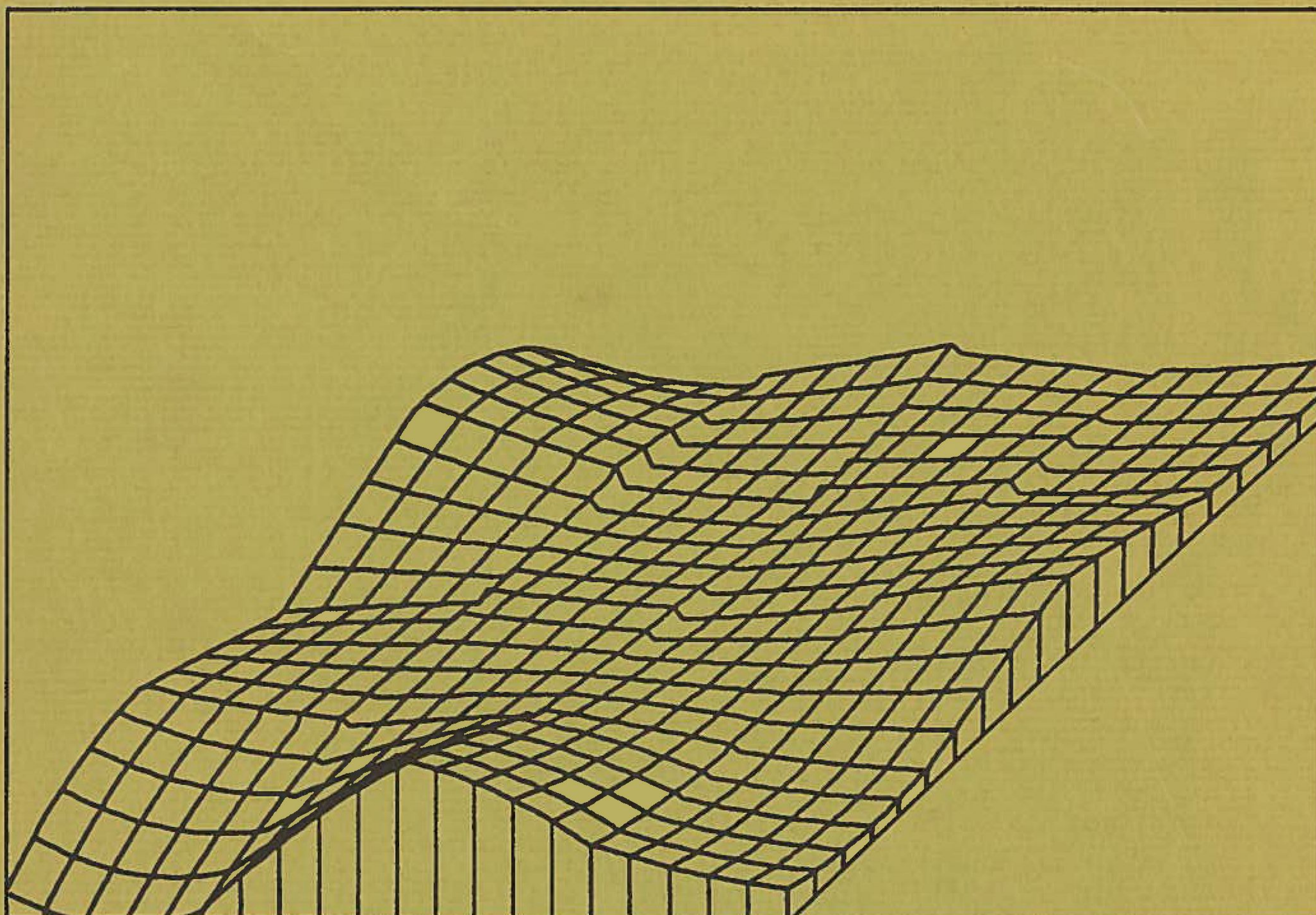

ECOMAP
A COMPUTER PROGRAM FOR MAPPING ECOLOGICAL DATA

Charles D. Bonham

Range Science Department

Science Series No. 9

Colorado State University

AUGUST 1971

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INTRODUCTION

In ecological disciplines, the major objectives for using operations research include: evaluation, optimization, and control of manipulative processes. Ecological evaluation problems arise, for example, in the assessment of availability of vegetation resources while optimization problems arise in the development of a management plan to use the vegetative resource. Control problems arise when consideration is given to the efficient application and successful operation of a management scheme designed especially for the ecological system. Major disciplines which are applicable to the solution of these problems include: ecology, statistics, systems analysis, and operations research. Computer mapping techniques have proven useful in solving problems existing in several disciplines and can be used for ecological evaluation purposes, determination of optimization procedures, and to control management procedures in ecological systems. To date, these computer mapping techniques have not been applied extensively to the study of ecological systems.

Computer mapping refers to presenting displays of response surfaces or contouring ecological variables of interest. These procedures are analagous to drawing in elevation contour lines for topographic maps and can be used to display ecological variables by levels over a particular geographic region. Specifically, we may be interested in studying the above ground standing crop and its characteristics over a given area. The use of computer contouring techniques is one approach that is available for this purpose. This procedure is useful for obtaining samples from a region and interpolating between each of the sample points.

Contours can be made of data for the geographical region of interest. Therefore an interpolation model is needed for mapping ecological data over a geographical region of interest. That is, we are interested in the geographical distribution and relative quantity of the ecological variable with respect to an x,y coordinate system.

The state of ecological technology is such that theory is seldom sufficiently complete to describe ecological variables quantitatively. Moreover, the spatial distribution of ecological variables in the system is little understood. Furthermore, little is known about interrelationships of variables that operate within a given ecological system. Consequently, a high degree of empiricism tends to prevail in quantitative ecology and related mathematical modeling.

Regression models are often used as a practical method for expressing a multiplicity of observations in a functional form which is suitable for manipulation by computer.

The classical approach to regression analysis is demonstrated by use of a one dimensional problem in which some relation

$$y = f(x) \tag{1}$$

is sought between the variable x and some response variable y dependent upon it. For example, x may be a critical variable such as precipitation, that is useful in predicting y, the yield of biomass per square kilometer. Observations are then made on a number N of (x,y) pairs and the theory of least squares is applied to obtain parameter estimates for the equation. A polynomial is the type of equation most generally used in such analyses. An example is

$$y = \beta_o + \beta_1 X + \beta_2 X^2 + \ldots + \beta_q X^q, \; q \leq N \tag{2}$$

Terms are added to the polynomial equation until a sufficiently good fit is obtained to satisfy some criterion. Ordinarily, a test based on the residual sums of squares is used to indicate that a significant amount of the y variation has been accounted for. However, two-dimensional mapping or contouring requires that the procedure use a slightly different form which includes the third variable. The generalized function is such that

$$Z = f(x,y) \tag{3}$$

where Z is the response variable and x and y are coordinate values for two orthogonal directions. Thus, computer mapping or contouring can be accomplished. This generalized function clearly indicates that a study could be made of a variable (Z) with respect any two orthogonal variables (x,y) which can be geographical position indicated by a pair of values. Another example is the use of a randomized block design where x is the block effect and y is the treatment effect and Z is the response variable.

Once the functional form is found which adequately expresses a relationship between the response variable, and the independent variables over a particular region of interest then the variance of that particular response variable can be derived. The generalized variance function for polynomials is expressed as:

$$V(Z) = \sum_{i=o}^{q} V(\beta_i) \; (X^i)^2 \quad , \qquad q \leq N \tag{4}$$

The variance of a function is used to study the variability of a variable with respect to another variable or set of variables.

The need to study ecological variability over a geographical region should be established so that a clear understanding of computer mapping and the scope of its application can be developed. The range of values that occurs for an ecological variable of interest may be indicated by the solution of the variance function. Therefore, it may be of major concern to find a region, for example within a given area where the variable being studied varies the most or the least.

Phytosociological mapping could also be carried out using a functional relationship of species with respect to their geographical position. In this way ecological mapping would essentially be a reconstruction of the vegetation elements over a particular region. Phytosociological mapping via computer has not been accomplished with great success to date, but may be very useful in the near future. The use of simulation procedures would enable a study to be made of the vegetation structure over a particular region. Furthermore, computer mapping would permit an examination to be made of phytosociological relationships among species with respect to their geographical location. Additionally, these procedures would be useful for analyses of vegetation patterns over a particular region of interest.

Methods And Approach To The Problem

In order to map ecological components for a region of interest, a mathematical representation of the ecological variable must be made. However, it has already been pointed out that the mathematical aspects of ecology has not yet advanced beyond the state of empiricism. Furthermore, ecological theory is not sufficiently complete to describe the distribution of ecological components geographically. Therefore, the basic approach taken here is one in which a regression model is used.

Under the assumptions of regression analyses, the best model is the one that yields the least residual sums of squares. It should be recognized that residual sums of squares are available only at points in the X space from which observations are made. Consequently, residual sums of squares may have little relation to the accuracy of the model at interpolation points in the X space where observations are not available. Yet, interpolation is exactly what is needed to map ecological variables over a region without extensive sampling. Commonly a mathematical model is selected and observed data are used to estimate the model parameters. The predictive model is then used to interpolate between sample points. It follows that additional data must be obtained from the interpolated region to evaluate the accuracy of the model. Thus, the regression approach can be evaluated for usefulness in mapping ecological characteristics over geographical regions of interest. In particular, if a linear model is used then statistical analysis of the data can be conducted easily.

There are non-statistical aspects of regression models which are important and have bearing on the justification for using them in computer mapping. In elementary mathematics, the domain and the range of a function is defined by ordered pairs of numbers. That is

$$D = (x_1, x_2 \ldots, x_n)$$

$$R = (y_1, y_2 \ldots, y_n)$$

The domain of a function is some interval along a line and any function that is defined on the interval is mathematically legitimate as long as it contains the original observations as a sub-set. Strictly speaking, the function is undefined for all other values of x and y. However, additional ordered pairs can be added to the set

$$f = (x_1, y_1), (x_2, y_2) \ldots, (x_n, y_n) \qquad (5)$$

which provides an extension (specified as f*) of the function. It follows that this is essentially an interpolation process, or an extension of the observational data which has been performed by adding additional ordered pairs to the set. The only function that is actually known consists of a set of (x,y) pairs obtained by sampling.

For an extension of an observational function to be valid, it must actually be capable of representing the relationship between the dependent and the independent variables. In the present consideration, the selected function must be capable of representing the ecological component over the (x,y) geographical space. This requirement, if not met, does not preclude the use of a selected function as a starting point in developing an acceptable model. It may be that the relationship between the response variable and the independent variables are of little or no interest. Yet, as long as the function does an acceptable job of describing the variable in the sense of predicting or reproducing the data, then it is justifiable to use it. To date, no function has been developed which describes ecological variables previously discussed.

Sets of values for (x,y) obtained for the development of ecological mapping procedures were those in which sample data were acquired according to Figure 1. One hundred stands from a grassland site (10m x 10m) were subsampled for several vegetation and soil characteristics (Bonham, 1969 and 1970). An interpolation process was needed to fill in the areas between the sample points. In order to develop a procedure for mapping ecological variables with respect to a geographical region of interest, an orthogonal set of polynomials having x and y components was chosen to describe the 3-dimensional space. The x and y components are needed

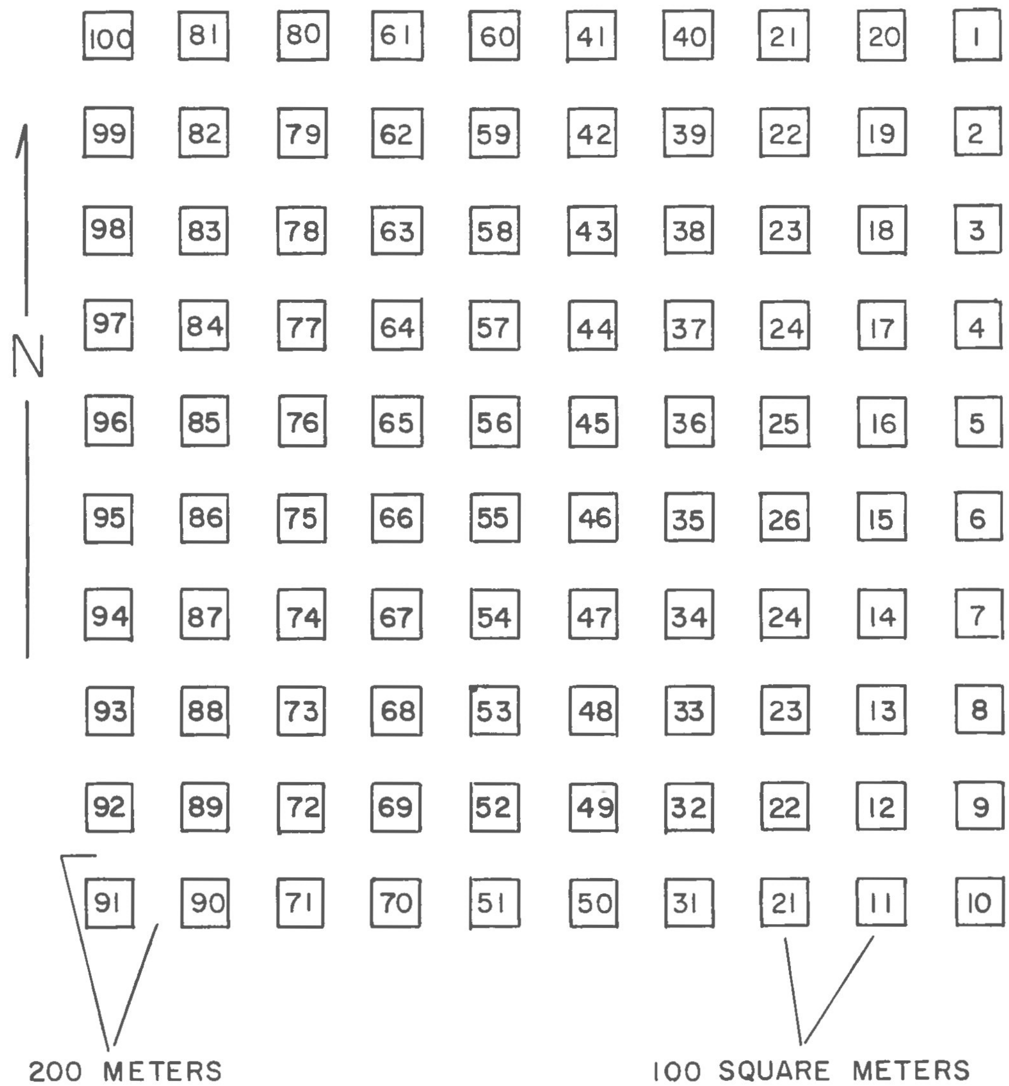


Figure 1. Field sampling design for stands (10m x 10m) which were placed 200 meters apart.

to represent geographical coordinates. The generalized form of the selected model is

$$Z = \beta_o + \beta_1 X + \beta_2 Y + \beta_3 XY + \beta_4 \ (3X^2-2) + \beta_5 \ (3Y^2-2) +$$

$$\beta_6 Y \ (3X^2-2) + \beta_7 X \ (3Y^2-2) \qquad (6)$$

A computer program was developed and combined with several subroutines for use in Ecological Mapping (ECOMAP). Two versions were developed in order to display results either digitally by a line printer or by microfilm plotter. Only the latter version is described here.

Data processing procedures for computer mapping of ecological variables are outlined in Figure 2. Step 1 indicates that a number of sub-samples have been taken at a particular sampling point in the (x,y) space. Means of ecological variables from the sampling points were then calculated and used as input for the computer mapping program.

The selection of a particular set of values to be used step-wise in the ECOMAP routine has been studied in detail and may vary according to particular needs of the user (Figure 2). A design region is best described as the region (or sub-region) over which the model will be applied to obtain estimates of the model parameters. An example of a particular design system is illustrated in Figure 3. The upper left hand corner of Figure 3 illustrates a 3 x 3 design region which is only one of many possible combinations. A 3 x 3 grid system has been found to give more efficient estimates of the variance of grassland components than either a 3 x 4, a 4 x 4, or a 3 x 5 grid system (Bonham, 1970).

The procedure used for computer mapping involves several steps before interpolation is actually carried out. Once a particular area grid system has been selected, then the sampling region is divided into

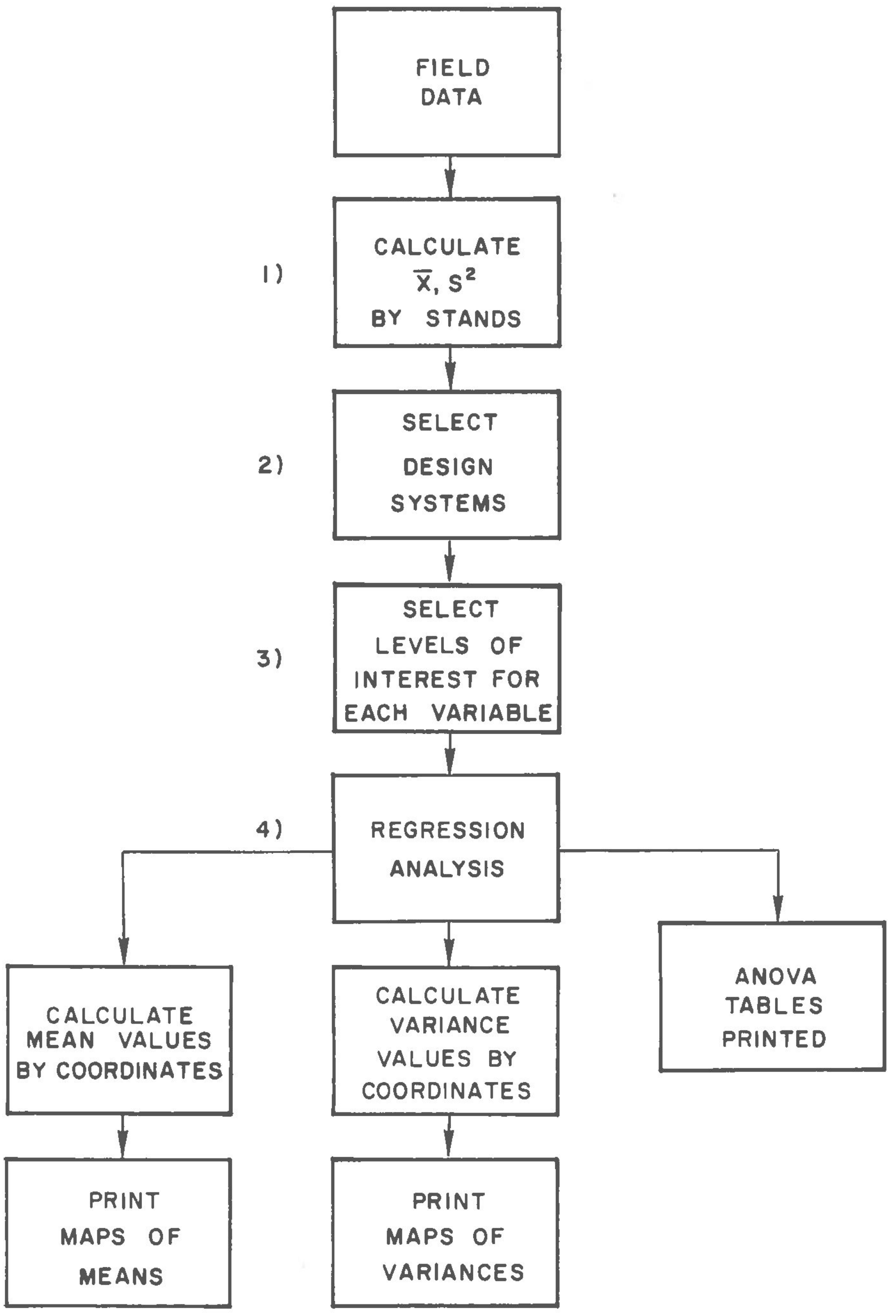
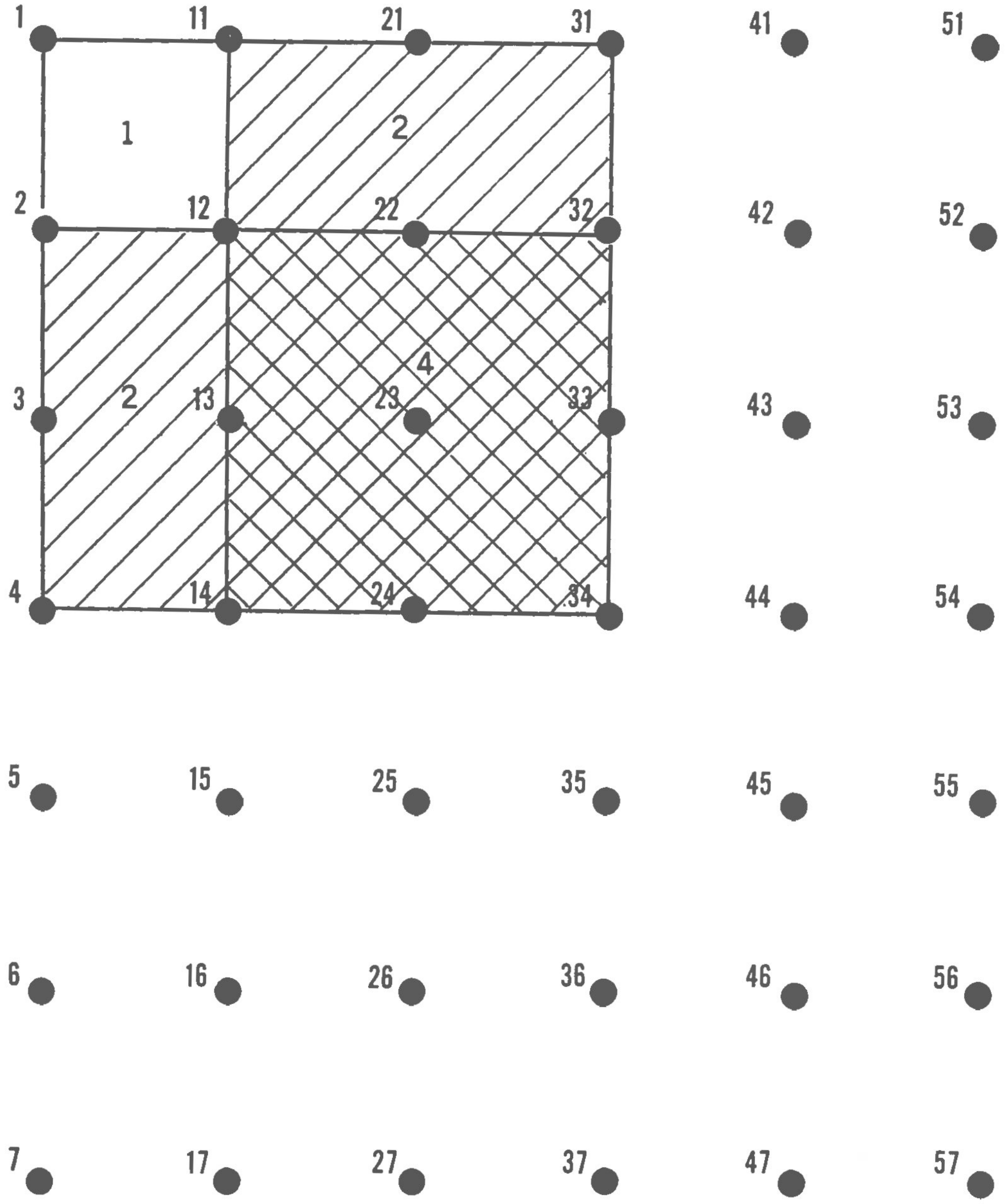


Figure 2. Data processing procedures for computer mapping of ecological variables.



set	points
k	12,13,14,22,23,24,32,33,34
k+1	13,14,15,23,24,25,33,34,35
k+2	14,15,16,24,25,26,34,35,36

Figure 3. Grid system formation procedures for mapping of ecological variables over geographical coordinates.

the smaller grids (in this case a 3 x 3) (Figure 3). The model parameters listed in equation (5) are then estimated using the data from this particular sampling region. By deletion of a row from the top of the design system and addition of a row at the bottom of the system, a number of different estimates of the response variable at a given point for (x,y) can be obtained in addition to a number of estimates for the model coefficients. That is, the first 9 points have as their centroid sample point 12 which is identified in the x,y space. The next 9 points have as their centroid sample point 13, also identified in the x,y space. This process is continued and gives a number of estimates of the model coefficients, as well as that of the response variable.

In this procedure, local surface fitting is applied to each of the sub-sets of the field data. Figure 3 illustrates that the process has been in progress for k-1 steps. At the k^{th} step, the operation involves the sample points: 12, 13, 14, 22, 23, 24, 32, 33, 34; with point 23 being the centroid (x,y). At k + 1 steps the sample points are respectively 13, 14, 15, 23, 24, 25, 33, 34, and 35. At each of these steps, 9 data points are used to estimate the coefficients of equation (5). The estimate of these parameters of the model are then used to generate a finer grid and to interpolate for the response variable over the entire geographical region. In this way, one estimate of the response variable, Z, and its associated variance, $\sigma^2_{Z.XY}$ is obtained for the four corners of the study region, two estimates are obtained for all the remaining border areas, and four estimates are obtained for all the interior areas (Figure 3).

An analysis of variance table for regression is printed out for each centroid, allowing an evaluation of the model to be made for each sub-region. Averages of interpolated points are obtained along with their variances which are then contoured on microfilm.

If all the data in the (x,y)-space are normalized with respect to the centroid of each respective (x,y) then the variance of the predicted response variable, Z, is the same for each 9-point grid in the study area. That this is so can be seen by observing the normalized values for x and y. Each 3 x 3 grid will have values as follows:

-1.0	0.	1.0
-1.0	0.	1.0
-1.0	0.	1.0

for values of x and:

1.0	1.0	1.0
0.	0.	0.
-1.0	-1.0	-1.0

for values of y. These values will be maintained for each 3 x 3 design region used if each (x,y) is normalized with respect to its centroid. This procedure is necessary for the components of equation (5) to be orthogonal to one another.

The inverse matrix $(X'X)^{-1}$ will be the same for each sub-region which in turn gives the same variance for the estimated model parameters. Since x and y are assumed to be constant (that is their variances equal zero), the variance of Z (estimated by $\hat{Z}$) is a function of the variance of the parameters only. That is,

$$V(Z) = V(\beta_o) + V(\beta_1)X^2 + V(\beta_2)Y^2 + V(\beta_3)(XY)^2 + V(\beta_4)(3X^2-2)^2 + V(\beta_5)(3Y^2-2)^2 + V(\beta_6)\{Y(3X^2-2)\}^2 + V(\beta_7)\{X(3Y^2-2)\}^2 \qquad (7)$$

and replacing $V(\beta_i)$ with $V(\hat{\beta}_i)$ in equation (6) will yield $V(\hat{Z})$. Since the constant multiplier, σ^2, was omitted the variance of the response variable is relative. Thus, all predicted or interpolated variables will have variance maps which are identical for each 3 x 3 grid.

ECOMAP was written in Fortran IV and is ready for use on most computers using a Fortran IV compiler. However, the microfilm plotter sub-routines are specific to the CDC 6400 and may cause some problems on other systems.

Discussion

The program ECOMAP has been used in studies of ecological variables and their respective variations for a geographical area. The model used as an example in equation (5) interpolates values quite accurately for measures of density (number of individuals per unit area), cover, and above-ground biomass of common plant species. Rare or infrequent species restricted to micro-habitats will usually not be detected as often in equally spaced sampling and are not as accurately mapped. Unequal sample spacing can be used in ECOMAP to overcome this problem, but this requires more input data and, furtheremore, efficiency is sacrificed elsewhere. That is, equally spaced sample points occur on intersections of the lines dividing design regions and can be used in a maximum of nine design regions. In contrast, unequal sample points which do not occur on these intersections can be used only four times in different design regions (Figure 3). This trade-off must be considered before field sampling is carried out.

Figure 4 illustrates a response surface generated from equation (5) for the cover of Bouteloua chondrosoides expressed as a percent of ground covered. Figure 5 illustrates contour slices taken from the response surface of Figure 4. Contour levels can be specified as input or automatically calculated as described in Appendix A. A bisectional view

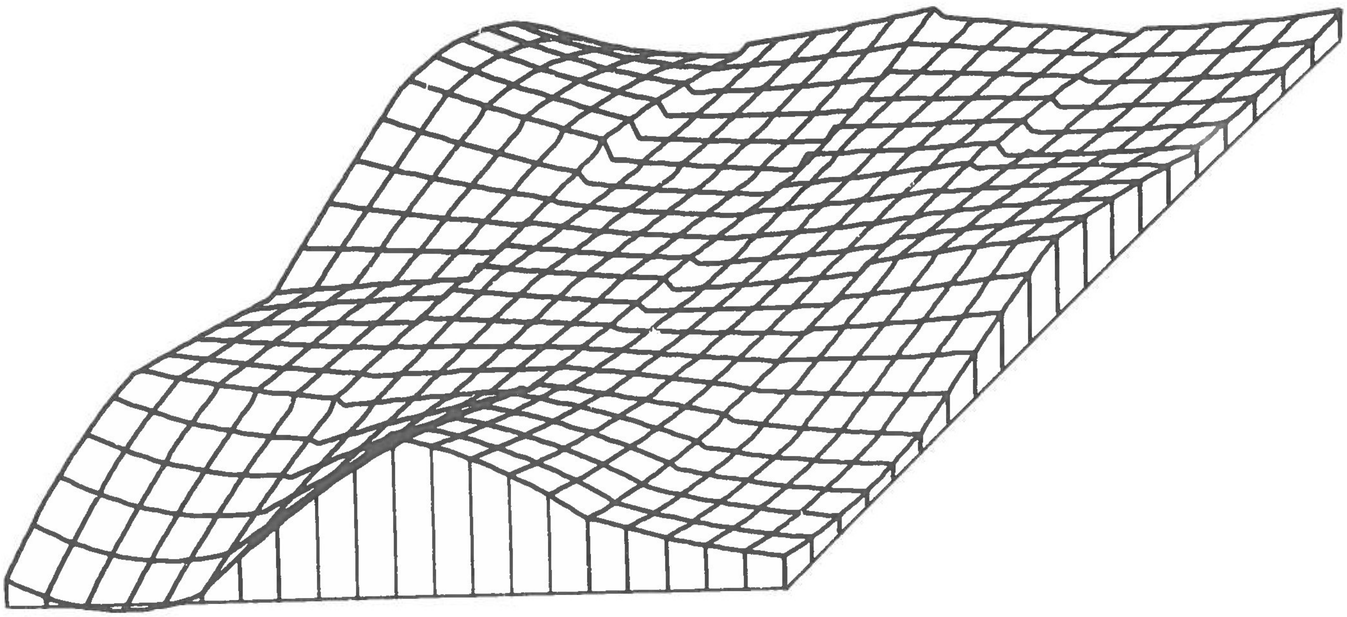
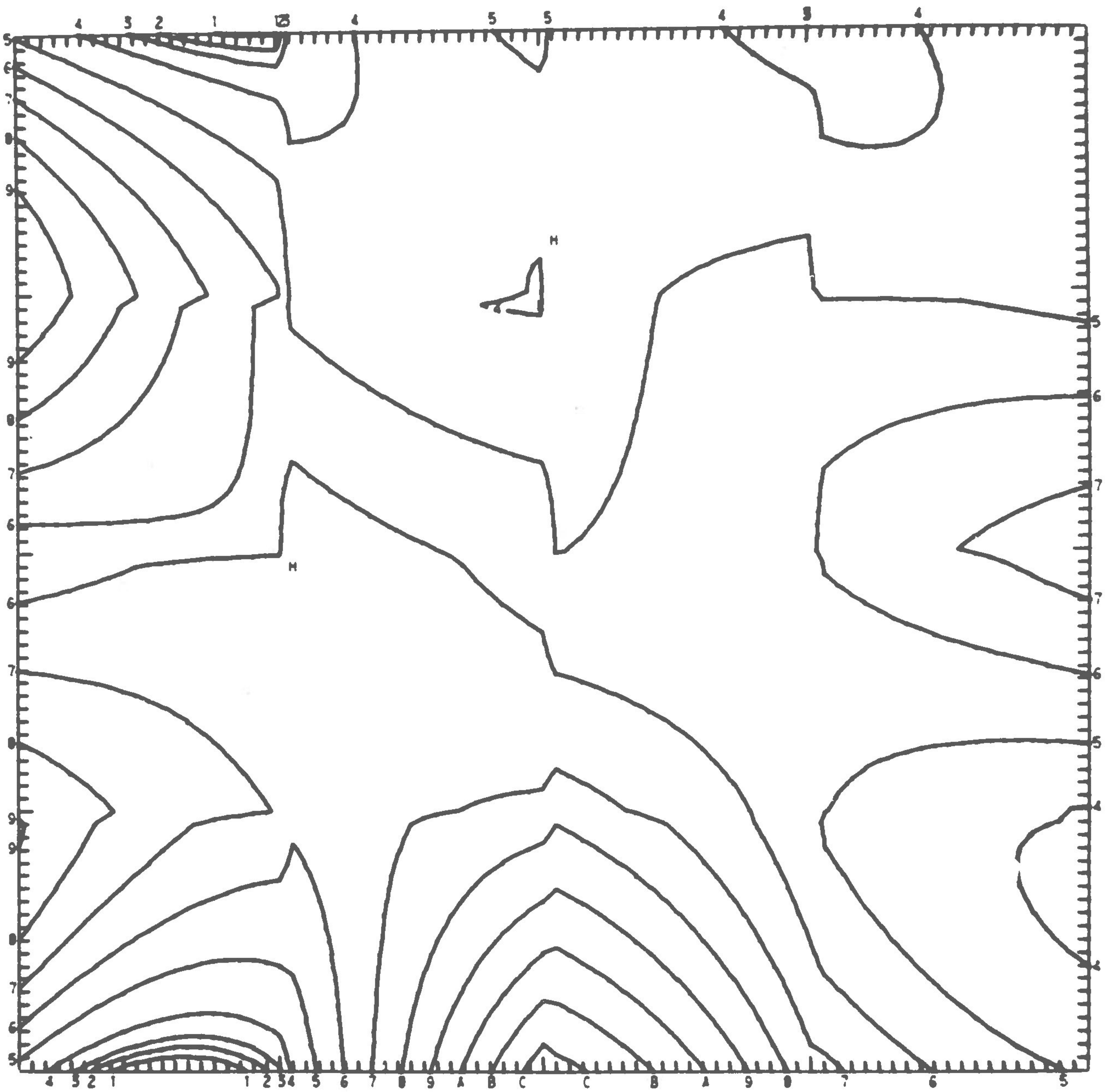


Figure 4. Bouteloua Chondrosiodes Cover (5 x 5, 1969).

CONTOUR
SYMBOL VALUES

1 =0.
2 = .50000E+00
3 = .10000E+01
4 = .20000E+01
5 = .40000E+01
6 = .60000E+01
7 = .80000E+01
8 = .10000E+02
9 = .12000E+02
A = .14000E+02
B = .16000E+02
C = .18000E+02
D = .20000E+02
E = .22000E+02
F = .24000E+02
G = .26000E+02
H = .28000E+02
I = .30000E+02
J = .32000E+02



CONTOUR VALUES IN PERCENT

THE MAP BOUNDARIES ARE x =	0.000 TO x =	1600.000	
AND y =	0.000 TO y =	1600.000	
EACH TICK MARK REPRESENTS	20.00000		IN THE x DIRECTION
	20.00000		IN THE y DIRECTION

Figure 5. Bouteloua Chondrosiodes Cover (5 x 5, 1969).

(or profile) of the response surface at any point selected by the user is illustrated in Figure 6.

In particular, two methods can be used to study statistical variations. One obtains a relative estimate of σ^2 calculated as

$$\mathrm{var}\ (\hat{z}) = x'\ (x'x)^{-1}\ x \tag{8}$$

This is contrasted to the estimate of an absolute variance obtained by

$$\mathrm{var}\ (\hat{z}) = x'\ (x'x)^{-1}\ x\ \ \sigma^2 \tag{9}$$

Unless σ^2 is actually known, an estimate of it must be used in equation (8). The estimate of σ^2, s^2, can be obtained from the model and the data values by the formula

$$s^2 = \sum^{n} (z_i - \hat{z}_i)\ /n-p \tag{10}$$

where $\hat{z}_i$ is obtained from the model and p is the number of parameters estimated in the model. Equation (8) can be used to estimate confidence intervals for a variable such as standing crop, soil pH, etc. However, the usual precautions of using regression models also apply here.

Figure 3 and the method of interpolation suggests that estimated variances for a particular point involves another constant, C. The value of C for all corners is 1, for remaining border areas is 2 and for all interior points is 4. Each set of 9 points gives one estimate of each parameter and all 3 x 3 grids adjacent to border areas provide two estimates of the response variable in interpolated areas unless the design is at one of the corners. In this case, only one estimate is available for interpolation. All interior regions of 3 x 3 grids provide four estimates of each point. The use of a constant then is necessary since an averaging process is used to estimate Z, the ecological variable.

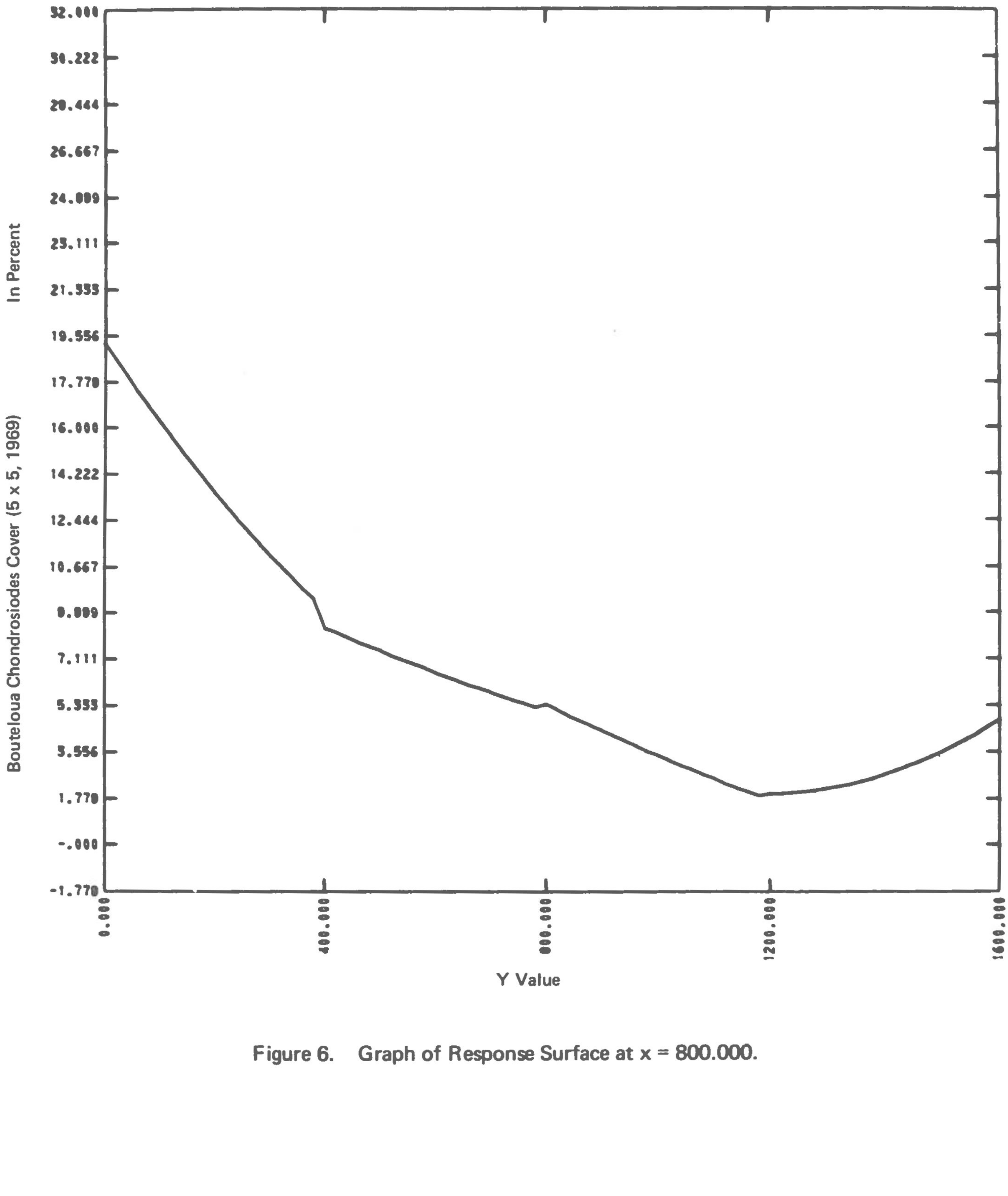


Figure 6. Graph of Response Surface at x = 800.000.

Thus, the variance of an individual point becomes

$$\mathrm{Var}\ (\hat{z}) = \frac{1}{C^2}\ x'\ (x'x)^{-1}x\ \ \sigma^2 \qquad (10)$$

The estimate of σ^2, s^2, can be used in equation (10) if σ^2 is unknown. Unless variances are of particular interest for computing confidence intervals, relative variances are just as useful for comparative purposes. Thus, equation (7) is used in ECOMAP and maps of relative variances are produced for further study. Figure 7 illustrates a relative variance response surface of percent cover of *Bouteloua chondrosoides* for the 1,000 acre study area. This figure illustrates some well-known statistical design concepts concerning variances. Smaller variances occur where more sample points are used toward the center of the design and larger variances occur toward edges or corners of treatment levels. Figure 8 illustrates contour levels of the variances of cover for the response surface of Figure 7. These maps are useful in the determination of areas of extreme ranges in values (large variance) or areas of homogeneity (low variance). Relative variances used in Figures 7 and 8 obscure this type of interpretation in detail. A larger grid (more points) may be necessary to obtain reasonably accurate maps of ecological variables within a specified area. Thus a preliminary study may be needed to determine grid size and shape for an optimum sampling design system for specific ecological variables.

The shortcomings of this mapping procedure are found mostly in the model as expressed in equation (5). Only one degree of freedom is available for estimating $\sigma^2_{Z.XY}$ from the ANOVA table. This is not serious if the main objective is to predict and not to test hypotheses about the

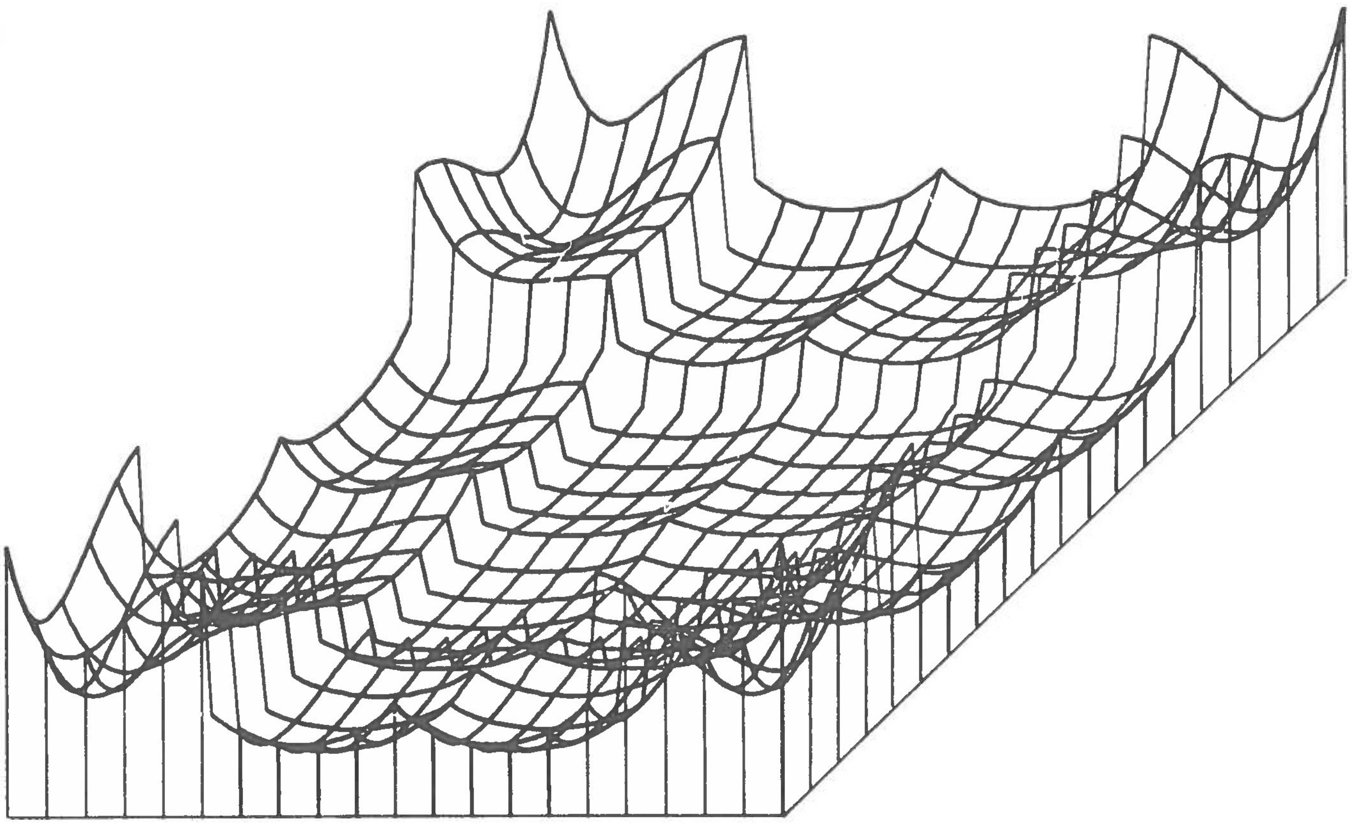
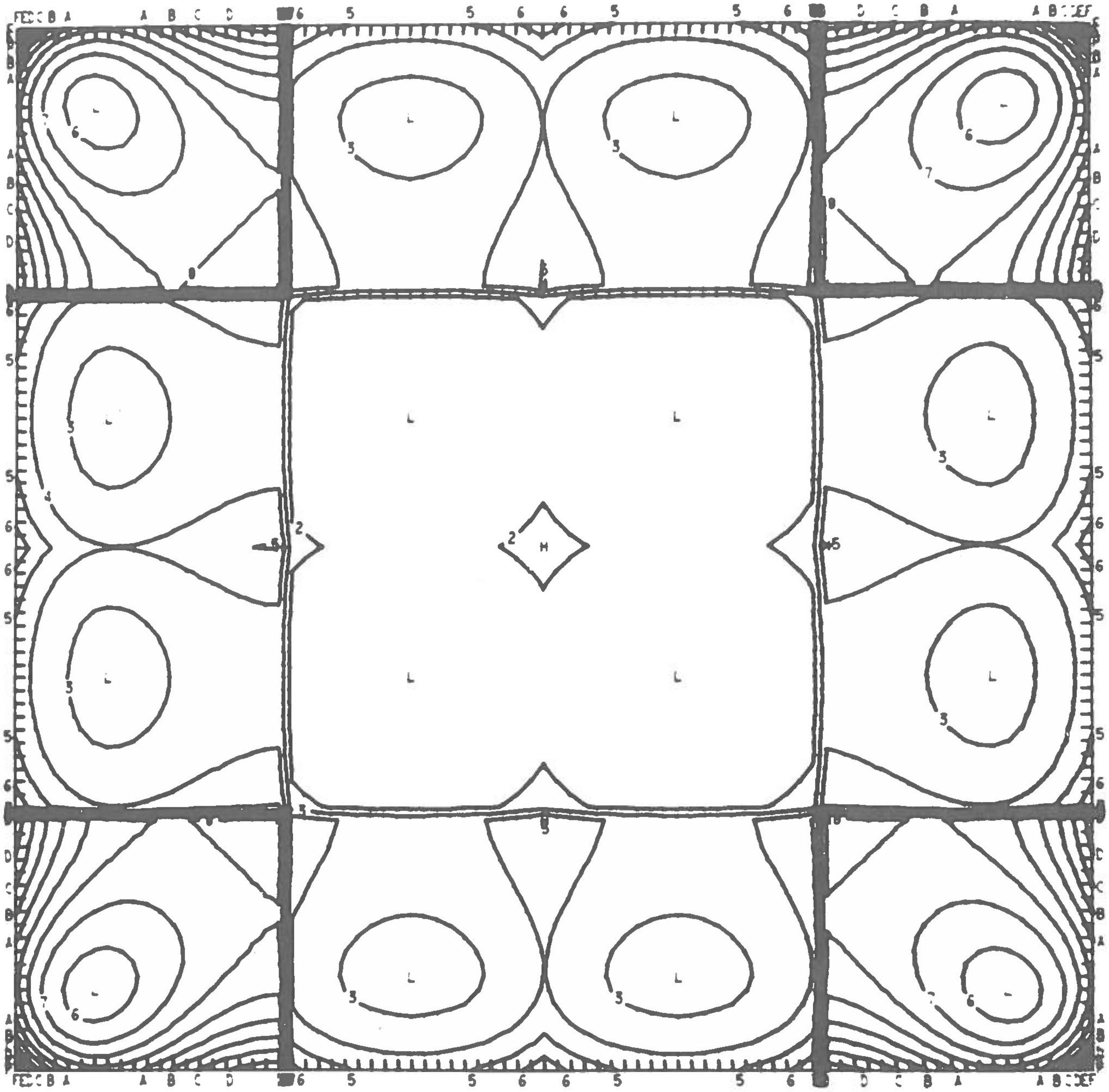


Figure 7. Bouteloua Chondrosiodes Cover Variance (5 x 5, 1969).

CONTOUR
SYMBOL VALUES

1 = .11220E+00
2 = .17363E+00
3 = .23506E+00
4 = .29649E+00
5 = .35792E+00
6 = .41935E+00
7 = .48078E+00
8 = .54221E+00
9 = .60364E+00
A = .66507E+00
B = .72650E+00
C = .78793E+00
D = .84936E+00
E = .91079E+00
F = .97222E+00



CONTOUR VALUES IN PERCENT
THE MAP BOUNDARIES ARE x = 0.000 TO x = 1600.000
AND y = 0.000 TO y = 1600.000
EACH TICK MARK REPRESENTS 20.00000 IN THE x DIRECTION
20.00000 IN THE y DIRECTION

Figure 8. Bouteloua Chondrosiodes Cover Variance (5 x 5, 1969).

model. The amount of total variation accounted for, R^2, is not as useful for evaluating the capability of the model when all but one data point is used in estimating model parameters. Another deficiency in the model is the fact that a polynomial can take on quite divergent values ranging from meaningless negative values to large positive values which cannot be interpreted. Otherwise, a three-dimensional model of the type in equation (5) appears to be adequate. Ecological responses are functions of their geographical location which is an integrator of all environmental variables and these responses need to be expressed in terms of coordinate function(s). ECOMAP is one approach to the problem.

Acknowledgements

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APPENDIX A

ECOMAP DESCRIPTION

AND LISTING

INPUT DATA FOR PROGRAM ECOMAP

Card Set	Column	Format	Variable
1	3	I1	NCI, Flag indicating coordinate system used. if = 0, indicates rectangular, if ≠ 0, indicates polar. (If polar coordinates were used in sampling).
	4-6	I3	NJ, the total no. of columns of sample points.
	7-9	I3	NA, the no. of basis functions to be used in the model. These must correspond to those in Sub-routine Basis.
	10-17	F8.0	DX, Sample spacing normalization constant in the x-direction. (The average spacing between sample points in the x-direction).
	18-25	F8.0	DY, Sample spacing normalization constant in the y-direction.
	26-33	F8.0	SF, the area on the xy plane represented by one interpolation point.
	34-41	F8.0	DTF, response data conversion flag. if = 0, no conversion; if = 1, data multiplied by CF
	42-49	F8.0	CF, response data conversion factor.
2	1-5	I5	IN(1), Number of sample points in column 1.
	6-10	I5	IN(2), Number of sample points in column 2.
	------------------------repeated NJ times		

Card Set	Column	Format	Variable
3	1-8	F8.0	XMINT, minimum x boundary of the total interpolation region.
	7-16	F8.0	XMAXT, maximum x boundary of the interpolation region.
	22-24	I3	JTN, the no. of interpolation points in the x direction.
	25-32	F8.0	YMINT, minimum y boundary of the total interpolation region.
	33-40	F8.0	YMAXT, maximum y boundary of the interpolation region.
	46-48	I3	ITN, the no. of interpolation points in the y direction.
	49-56	F8.0	XINC, increment between successive design, systems in the x direction.
	57-64	F8.0	YINC, increment between successive design systems in the y direction.
4	-----contains 1 card for each sample point as follows: (May be computed and punched by ECOGRID which is listed)		
	1-8	F8.0	X(I,J) x coordinate of each sample point.
	9-16	F8.0	Y(I,J) y coordinate of each sample point.
	17-24	F8.0	XMINI (I,J) x-coordinate limits
	25-32	F8.0	XMAXI (I,J) of the design region centered on each sample point.
	33-40	F8.0	YMINI (I,J) y-coordinate limits
	41-48	F8.0	YMAXI (I,J) of the design region centered on each sample point.

Card Set	Column	Format	Variable
5	1-80	8A10	IFM, the format for reading the response value at each point in one column. (Must be a floating point format)
6	??	IFM	Z (I,J), the response value at each point.
7	1-50	5A10	TITLE, holerith identifier of the response being mapped.
8	1-10	F10.0	ZMIN Convenient minimum and
	10-20	F10.0	ZMAX Maximum response values
	21-25	I5	NCZ, the no. of contour levels on the response contour plot. ---if positive, contour levels are calculated using ZMIN, ZMAX, and NCZ. ---if negative, contour levels are read in.
	26-30	I5	NCV, the no. of contour levels on the response variance contour plot.
	31-35	I5	NPLOT, the no. of XZ and YZ plots requested
	36-40	I5	N1X The no. of sample points
	41-45	I5	N1Y between lines plotted on the 3d response surface plot in the x and y directions. 3 = 31 lines for 91 x 91
	50	I5	NVAR, a flag for requesting variance plots. ---if = 0 variance plots provided. ---if = 1 variance plots not provided.

Card Set	Column	Format	Variable
9	-----Optional, used only when NCZ is negative		
	1-8	F8.0	CLVL (1), value of contour level no. 1.
	9-16	F8.0	CLVL (2)
	17-24	F8.0	CLVL (3)
	------------------------------------repeated -ncz times.		
10	-----Optional, used only when NPLOT is greater than 0.		
	1-8	F8.0	PLOT (1), --- if positive = the X value of a requested yz plot. ---if negative = the Y value of a requested xz plot.
	8-16	F8.0	PLOT (2)
	------------------------------------repeated nplot times.		
11	1-50	5A10	TITLE, the holerith identifier of the response being mapped.
	51-60	A10	ZUNITS, the holerith identifier of the units of the response.
	61-70	A10	XUNITS, the holerith identifier of the x and y coordinate units.
12	-----Optional, used only when NVAR is equal to 0		
	1-50	5A10	TITLE, the holerith identifier of the response variance.
	51-60	A10	ZUNITS, the holerith identifier of the response variance units.
	61-70	A10	XUNITS, the holerith identifier of the x and y coordinate units.

REPEAT CARD SETS 5 - 12 for as many data sets as are to be run.

Additional Details

1. The sample point coordinates and response values at each point must be read in in the following order, based on the diagram below: (The numbers by the dots are the order)

XMIN, YMAX		XMAX, YMAX	
.1	.5	.9	.13
.2	.6	.10	.14
.3	.7	.11	.15
.4	.8	.12	.16
XMIN, YMIN		XMAX, YMIN	

   In other words, the data is read in column by column, starting with the left column, and within each column starting at the first point.

2. IFM must be a real (floating point) format and specifies the reading in of each point in one column.

3. The design system must be set up so there is no design region centered about the first or last point in any row or column.

   ---if there is no design region about a point, merely leave XMINI, XMAXI, YMINI, and YMAXI blank.

LIMITS FOR ECOMAP

No. of sample points $\leq$ 100 (10 x 10 grid)

No. of interpolation points $\leq$ 8281 (91 x 91 grid)

No. of basis functions $\leq$ 9

No. of points within a design region $\leq$ 15

No. of requested YZ and XZ plots $\leq$ 10

No. of contour levels $\leq$ 25

NOTES ON ECOMAP

To increase the limit of the number of points within a design region, increase the dimensions of XS, YS, ZS, and the second dimension of DTRAN to the desired limit in all common/MATOD/statements. In DESYS it will be necessary to increase the dimensions of XM, YM, XMP, and YMP to the desired limit.

TO ADD EXTRA SAMPLING POINTS TO A GRID WITH EQUAL SAMPLE SPACING

1. Read in the x and y {X(I,J) and Y(I,J)} coordinates with the first column to the left of the point.
2. Within that column read all points in in order from north to south.
3. Read in the response values Z(I,J) in the same order.

STEPS FOR SETTING UP GRID SYSTEM FOR UNEQUAL SAMPLE SPACING

1. Plot all sampling points on a map.
2. Draw a rectangular boundary of the area to be contoured.
3. Decide on the increment between successive design regions in the x and y directions (the design regions must be equally spaced!!!)
4. Draw the lines dividing the design regions on the map. (There must not be more than 15 points within a design region).
5. Draw a dotted line down the middle of the left-most column on the grid before you.
6. Now each vertical line (including the dotted line) defines the boundaries of a column. The coordinates of each point are read in column by column from left to right, and in order from top to bottom within each column. Pick a sampling point near the center of each

design region and read in the x and y limits of the region with the coordinates of that sampling point. The sampling points picked for this purpose cannot be in the first or last column, or the first or last point in any column!!! For the points not used for reading in design region limits leave XMINI, XMAXI, YMINI, and YMAXI blank.

7. Read in the response values at each sample point in the same order as you read in their coordinates.

```
      PROGRAM ECOMAP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,FILMPL)
C*****OBJECTIVE**********************
C          THIS PROGRAM COMPUTES INTERPOLATED, REGULARLY SPACED Z VALUES
C     FROM A DATA SET OF SAMPLE Z VALUES.  IT DOES SO BY SELECTING SMALL
C     SUBREGIONS (DESIGN REGIONS) WITHIN THE REGION OF INTEREST, PER-
C     FORMING A 3 DIMENSIONAL REGRESSION (Z=F(X,Y)), AND CALCULATING
C     THE INTERPOLATION VALUES.  VARIANCE ESTIMATES ARE OBTAINED FOR
C     EACH POINT, THEN OVERLAPPING INTERPOLATION AND VARIANCE VALUES ARE
C     AVERAGED.  THE BASIS FUNCTIONS USED FOR REGRESSION ARE BOX,S
C     ORTHOGONAL POLYNOMIALS.
C
C*****SCOPE**********************
C          THE PROGRAM WILL ACCEPT EITHER A RECTANGULAR OR POLAR
C     COORDINATE SYSTEM, WITH EVENLY SPACED OR SCATTERED SAMPLE POINTS.
C     HOWEVER THE DESIGN REGIONS MUST BE OF EQUAL SIZE AND EQUALLY
C     SPACED.  THE PRESENT PROGRAM LIMITS THE USER TO 100 SAMPLE POINTS,
C     8281 INTERPOLATION POINTS, AND A DESIGN REGION CONTAINING 9 POINTS.
C
C*****VARIABLE LIST***************************
C        A(I) = COEFFICIENTS OF BASIS FUNCTIONS.
C        AVGMAT(I,J) = ARRAY OF INTERPOLATED RESPONSE VALUES.
C        BASE(I) = VALUES OF BASIS FUNCTIONS.
C        CF = RESPONSE VALUE CONVERSION FACTOR.
C        DINV = INVERSE OF SQUARE DESIGN MATRIX (XP*X)**-1.
C        DTF = RESPONSE DATA CONVERSION FLAG ---IF = 0, NO CONVERSION.
C                                           ---IF = 1, DATA MULTIPLIED BY CF
C        DX, DY = SAMPLE SPACING NORMALIZATION CONSTANTS (X AND Y
C             COORDINATES)
C        DXT,DYT = INTERVAL SPACING BETWEEN INTERPOLATION POINTS (X AND
C             Y COORDINATES)
C        IC = NO. OF POINTS WITHIN THE DESIGN REGION.
C        IN(J) = TOTAL NO. OF SAMPLE POINTS IN COLUMN J (J=1,2,.....NJ)
C        INVCK = INVERSION CHECK----IF MATRIX IS SINGULAR, = -1.
C                                 ----OTHERWISE = ORDER OF THE MATRIX.
C        ITN, JTN = TOTAL NO. OF INTERPOLATION POINTS IN THE Y AND X
C             DIRECTIONS RESPECTIVELY.
C        LINE = THE NUMBER OF THE LINE TO BE PRINTED ON THE OUTPUT PAGE.
C        NA = TOTAL NO. OF BASIS FUNCTIONS TO BE USED.
C        NC1 = DENOTES DATA IN POLAR COORDINATES IF UNEQUAL TO 0.
C        NF = TOTAL NO. OF BASIS FUNCTIONS USED.
C        NJ = TOTAL NO. OF COLUMNS OF SAMPLE POINTS.
C        PAGENO = NUMBER OF THE PRESENT OUTPUT PAGE.
C        SF = THE AREA ON THE XY PLANE REPRESENTED BY ONE INTERPOLATION
C             POINT.
C        TITLE = HOLERITH RESPONSE PARAMETER IDENTIFIER.
C        VAR(I,J) = ARRAY OF VARIANCE FOR THE INTERPOLATED RESPONSE.
C        X(I,J), Y(I,J) = X AND Y COORDINATES OF EACH RESPONSE VALUE Z(I,J)
C        XC, YC = X AND Y COORDINATES OF THE DESIGN SYSTEM CENTROID.
C        XIN, YIN = NORMALIZED COORDINATES OF AN INTERPOLATION POINT.
C        XINC, YINC = INCRIMENT (IN X AND Y DIRECTIONS) BETWEEN SUCCESS-
C             IVE DESIGN SYSTEMS.
C        XMINI(I,J), XMAXI(I,J) = X COORDINATE LIMITS OF THE DESIGN
C             REGION CENTERED ON THE I, J SAMPLE.
C        XMINT, XMAXT = LIMITS OF THE TOTAL REGION OF INTEREST (OVER
C             WHICH INTERPOLATION VALUES ARE CALCULATED) IN THE X DIRECTION.
C        XP, YP = COORDINATES OF AN INTERPOLATION POINT.
C        XS(IC), YS(IC) = COORDINATES OF ALL POINTS WITHIN THE DESIGN REGION.
C        YMINI(I,J), YMAXI(I,J) = Y COORDINATE LIMITS OF THE DESIGN
C             REGION CENTERED ON THE I, J SAMPLE.
C        YMINT, YMAXT = LIMITS OF THE TOTAL INTERPOLATION REGION IN THE
C             Y DIRECTION.
C        Z(I,J) = RESPONSE VALUE OF THE I, J SAMPLE.  VALUE OF -1
C             INDICATES MISSING DATA AT THAT SAMPLE POINT.
C        ZIN = INTERPOLATED RESPONSE.
C        ZS(IC) = RESPONSE AT EACH POINT WITHIN THE DESIGN REGION.
C****************************************************************
      INTEGER PAGENO
      DIMENSION X(10,10),Y(10,10),Z(10,10),XMINI(10,10),XMAXI(10,10),
     1 YMINI(10,10),YMAXI(10,10),IN(10),AVGMAT(91,91),IFM(8),A(9)
      DIMENSION TITLE(5)
      COMMON VAR(91,91)
      COMMON/MATOD/XS(15),YS(15),DTRAN(9,15),ZS(15)
      COMMON/ALL/NF,BASE(9),DINV(9,9),XC,YC,DX,DY,INVCK
      COMMON/MTOVR/XMNS,XPLS,YMNS,YPLS
      COMMON/LINENO/LINE,PAGENO,TITLE
      LOGICAL FIRST
C
C*****INPUT DATA
C  ---READ ARRAY SIZES, LIMITS, AND CODES.
    1 READ(5,201) NC1,NJ,NA,DX,DY,SF,DTF,CF,(IN(J),J=1,NJ)
      READ (5,203) XMINT,XMAXT,JTN,YMINT,YMAXT,ITN,XINC,YINC
C  ---READ GRID SYSTEMS
      DO 7 J=1,NJ
      NI=IN(J)
      READ(5,202) (X(I,J),Y(I,J),XMINI(I,J),XMAXI(I,J),YMINI(I,J),
     1 YMAXI(I,J),I=1,NI)
    7 CONTINUE
C  ---INITIAL CONDITIONS.
      CALL FRAME
    8 DO 102 KKK=1,91
      DO 102 KKKK=1,91
      VAR(KKK,KKKK)=0.
  102 AVGMAT(KKK,KKKK)=0.
      LINE=0
      PAGENO=0
C  ---FORMAT FOR READING FIELD DATA.
      READ(5,204)IFM
```

```
      IF(EOF(5)) 109,9
C  ---READ FIELD DATA.
    9 DO 10 J=1,NJ
      NI=IN(J)
      READ(5,IFM)(Z(I,J),I=1,NI)
      IF(EOF(5)) 109,10
   10 CONTINUE
C  ---READ DATA IDENTIFIER.
      READ(5,205) TITLE
C
C*****DATA CONVERSION IF REQUESTED.
      IF (DTF.EQ.1)11,12
   11 CALL DATA (Z,NJ   ,IN,CF)
   12 CONTINUE
C
C*****CALCULATE DISTANCE BETWEEN INTERPOLATION POINTS.
      DXT = (XMAXT-XMINT)/FLOAT(JTN-1)
      DYT = (YMAXT-YMINT)/FLOAT(ITN-1)
      FIRST=.TRUE.
      NJM=NJ-1
C
C*****PERFORM OPERATIONS ON ALL DESIGN REGIONS.
      DO 84 J=2,NJM
      NIM =IN(J)-1
      DO 83 I=2,NIM
      XMNS=XMINI(I,J)
      XPLS=XMAXI(I,J)
      YMNS=YMINI(I,J)
      YPLS=YMAXI(I,J)
      IF (XPLS.EQ.XMNS) GO TO 83
C
C*****FIND ALL POINTS WITHIN THE PRESENT DESIGN REGION, AND PRESERVE
C          THEIR COORDINATES AND RESPONSE VALUES.
      IC=0
      DO 41 JJ=1,NJ
      IF(X(1,JJ).GT.XPLS) GO TO 42
      IF(X(1,JJ).LT.XMNS) GO TO 41
      NI=IN(JJ)
      DO 40 II= 1,NI
      IF(Y(II,JJ).LT.YMNS) GO TO 41
      IF(Y(II,JJ).GT.YPLS) GO TO 40
      IF(Z(II,JJ).LT.0.) GO TO 40
      IC=IC+1
      XS(IC)=X(II,JJ)
      YS(IC)=Y(II,JJ)
      ZS(IC)=Z(II,JJ)
   40 CONTINUE
   41 CONTINUE
   42 NP=IC
      NF=NA
      IF(NP.LT.NA) NF=NP
C
```

```
C*****CONVERSION FROM POLAR TO RECTANGULAR IF NECESSARY.
      IF(NC1.NE.0) CALL CONVRT(NP)
C
C*****NORMALIZE POINTS IN THE DESIGN REGION AND COMPUTE DESIGN MATRIX,
C          SQUARE DESIGN MATRIX, AND ITS INVERSE.
      CALL DESYS(NP,FIRST)
      FIRST=.FALSE.
C  ---IF DATA IN POLAR, MORE CONVERSIONS.
      IF (NC1.NE.0) CALL DEFINE
C  ---IF DESIGN MATRIX SINGULAR, SKIP REGRESSION.
      IF (INVCK.LT.0)GO TO 83
C  ---COMPUTE COEFFICIENTS OF BASIS FUNCTIONS.
   50 DO 55 II=1,NF
      A(II)=0.
      DO 55 JJ=1,NP
      DO 55 K=1,NF
   55 A(II)=A(II)+DINV(II,K)*DTRAN(K,JJ)*ZS(JJ)
C
C*****REGRESSION.
      CALL REGRES(NP,SIGZXY,SIGZ,A)
C
C*****PERFORM CALCULATIONS FOR EACH INTERPOLATION POINT WITHIN THE DESIGN
C          REGION.
      MJT= (XMNS-XMINT)/(.99999*DXT) + 2.
      NJT = (XPLS-XMINT)/(.99999*DXT) +1.
      MIT= (YMAXT-YPLS)/(.99999*DYT) + 2.
      NIT = (YMAXT-YMNS)/(.99999*DYT) + 1.
      IF(MJT.EQ.2) MJT=1
      IF(MIT.EQ.2) MIT=1
      DO 82 IT=MIT,NIT
      YP = YMAXT - FLOAT(IT-1)*DYT
      DO 81 JT = MJT,NJT
      XP = XMINT +FLOAT(JT-1)*DXT
C  ---COORDINATE CONVERSION (FOR POLAR COORDINATES).
      IF(NC1.EQ.0) GO TO 70
      TIJ=57.2957795*ATAN2(YP,XP)
      L1=1
      L2=1
      IF(TIJ.GT.YMAXI(I,J)) L1=2
      IF(TIJ.LT.YMINI(I,J)) L2=2
      IF (YMAXI(I,J).LT.0.) 60,61
   60 GO TO (62,84),L1
   61 GO TO (62,81),L1
   62 IF (YMINI(I,J).LT.0.) 71,72
   71 GO TO (64,84),L2
   72 GO TO (64,82),L2
   64 RIJ=SQRT(XP**2+YP**2)
      IF(RIJ.GT.XMAXI(I,J)) GO TO 82
      IF(RIJ.LT.XMINI(I,J)) GO TO 81
   70 CONTINUE
      XIN=(XP-XC)/DX
```

```
      YIN=(YP-YC)/DY
C  ---BASIS COEFFICIENTS FOR PRESENT INTERPOLATION POINT.
   74 CALL BASIS(XIN,YIN)
      ZIN =0.
C  ---COMPUTE Z VALUE AT PRESENT INTERPOLATION POINT.
      DO 76  K=1,NF
      ZIN=ZIN+A(K)*BASE(K)
   76 CONTINUE
      AVGMAT(IT,JT)=AVGMAT(IT,JT)+ZIN
      SUM=0.
C  ---COMPUTE VARIANCE OF PRESENT INTERPOLATION VALUE.
      DO 78 IV=1,NF
   78 SUM=SUM+BASE(IV)*DINV(IV,IV)*BASE(IV)
      VAR(IT,JT)=VAR(IT,JT)+SUM
   81 CONTINUE
   82 CONTINUE
   83 CONTINUE
   84 CONTINUE
C
C*****AVERAGE OVERLAPPING INTERPOLATION VALUES AND VARIANCES.
      INCY=YINC/(.99999*DYT)
      INCX=XINC/(.99999*DXT)
      DO 103 J=1,JTN
      DO 103 I=1,ITN
      IF (I.LE.(INCY+1).O.I.GE.(ITN-INCY+1)) 104,105
  104 IF (J.LE.(INCX+1).O.J.GE.(JTN-INCX+1)) 103,106
  105 IF (J.LE.(INCX+1).O.J.GE.(JTN-INCX+1)) 106,107
  106 AVGMAT(I,J)=AVGMAT(I,J)/2.
      VAR(I,J)=VAR(I,J)/4.
      GO TO 103
  107 AVGMAT(I,J)=AVGMAT(I,J)/4.
      VAR(I,J)=VAR(I,J)/16.
  103 CONTINUE
C
C*****MAKE RESPONSE SURFACE AND CONTOUR PLOTS OF INTERPOLATED RESPONSE
C            AND VARIANCE.
      CALL PRINPLT(AVGMAT,ITN,JTN,SF,DXT,DYT,XMINT,YMAXT,XINC,YINC)
      GO TO 8
  109 CALL EXIT
  201 FORMAT(3I3,5F8.0/(15I5))
  202 FORMAT(6F8.0)
  203 FORMAT(2(2F8.0,5X,I3),2F8.0)
  204 FORMAT(8A10)
  205 FORMAT(5A10)
      END
```

```
      SUBROUTINE DATA (Z,NJ    ,IN,CF)
C           THIS SUBROUTINE MULTIPLIES INPUT DATA (Z(I,J)) BY A CONSTANT
C     CONVERSION FACTOR (CF).
C
      DIMENSION IN(10),Z(10,10)
      DO 10 J=1,NJ
      NI=IN(J)
      DO 10 I = 1,NI
      Z(I,J)= Z(I,J) *CF
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BASIS(X,Y)
C     COMPUTES VALUES OF ALL BASIS FUNCTIONS AT DEFINED POINT (X,Y).
      COMMON /ALL/NF,BASE(9),DUM(86)
    1 BASE(1)= 1.
    2 BASE(2)= X
    3 BASE(3)= Y
    4 BASE(4)= X*Y
    5 BASE(5)= 3.*X**2  - 2.
    6 BASE(6)= 3.*Y**2  - 2.
    7 BASE(7)= BASE(5)*Y
    8 BASE(8)= BASE(6)*X
    9 BASE(9)=BASE(6)*BASE(5)
      RETURN
      END
```

```
      SUBROUTINE DESYS(NP,FIRST)
C     CONSTRUCTS DESIGN SYSTEM MATRICES DEFINED BY ARRAY OF SELECTED
C     POINTS AND BASIS FUNCTIONS.   THEN COMPUTES INVERSE OF SQUARE
C     DESIGN MATRIX, AND PRINTS OUT THE RESULTS.
C
C*****ARGUMENT LIST************************
C         BASE(I) = VALUES OF BASIS FUNCTIONS.
C         DTRAN(I,J) = TRANSPOSE OF DESIGN MATRIX.
C         DX,DY = SAMPLE SPACING NORMALIZATION CONSTANTS.
C         FIRST = LOGICAL FLAG, IF TRUE, THIS IS THE FIRST TIME THIS
C              SUBROUTINE HAS BEEN CALLED.
C         LINE = LINE NUMBER FOR PRINTOUT.
C         NF = NO. OF BASIS FUNCTIONS USED.
C         NP = NO. OF POINTS IN THE PRESENT DESIGN REGION.
C         NPP = NO. OF POINTS IN THE LAST DESIGN REGION.
C         PAGENO = THE NUMBER OF THE PRESENT PRINTOUT PAGE.
C         X(I), Y(I) = COORDINATES OF ALL POINTS IN THE DESIGN REGION.
C         XC, YC = COORDINATES OF THE CENTROID OF THE DESIGN REGION.
C         XM(I), YM(I) = NORMALIZED COORDINATES OF ALL POINTS IN THE
C              PRESENT DESIGN REGION.
C         XMP(I), YMP(I) = NORMALIZED COORDINATES OF ALL POINTS IN THE
C              LAST DESIGN REGION.
C         XPXIN(I,J) = FIRST THE SQUARE DESIGN MATRIX, LATER THE INVERSE
C              OF THE SQUARE DESIGN MATRIX.
C*********************************************************************
      INTEGER PAGENO
      DIMENSION XM(15),YM(15),XMP(15),YMP(15)
      COMMON/MATOD/X(15),Y(15),DTRAN(9,15),DUM(15)
      COMMON/ALL/NF,BASE(9),XPXIN(9,9),XC,YC,DX,DY,NFI
      COMMON/LINENO/LINE,PAGENO
      LOGICAL FIRST
      IF(FIRST) NPP=0
      J=0
C
C*****COMPUTE DESIGN SYSTEM CENTROID
      SUMX=0.0
      SUMY=0.0
      DO 7 I=1,NP
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
    7 CONTINUE
      PN=NP
      XC=SUMX/PN
      YC=SUMY/PN
C
C*****COMPUTE NORMALIZED DESIGN SYSTEM.
      DO 28 I=1,NP
C  ---NORMALIZE SAMPLE POINT COORDINATES.
      XM(I)=(X(I)-XC)/DX
      YM(I)=(Y(I)-YC)/DY
      IF(FIRST) GO TO 19
C  ---IF THE LAST SET OF NORMALIZED X,Y COORDINATES WAS THE SAME AS THIS
C     SET CONTINUE, OTHERWISE RECOMPUTE XMP,YMP,BASE, AND DTRAN.
      A= ABS(XM(I)-XMP(I))
      B= ABS(YM(I)-YMP(I))
      IF(A.GE.0.0001.OR.B.GE.0.0001) GO TO 19
      J=J+1
      GO TO 28
   19 XMP(I)=XM(I)
      YMP(I)=YM(I)
      XB=XM(I)
      YB=YM(I)
   25 CALL BASIS(XB,YB)
      DO 27 L=1,NF
      DTRAN(L,I)= BASE(L)
   27 CONTINUE
   28 CONTINUE
C  ---IF PRESENT DESIGN MATRIX THE SAME AS THE LAST, RETURN.
      IF(J.EQ.NP.AND.NP.EQ.NPP) RETURN
      LINE=LINE+60
      CALL PAGE
      LINE=1
C  ---WRITE CENTROID LOCATION.
      WRITE(6,220) XC,YC
C
C*****COMPUTE SQUARE DESIGN MATRIX.
      DO 38 I=1,NF
      DO 37 J=I,NF
      IF(J.LT.I) GO TO 37
      XPXIN(I,J)=0.0
      DO 35 K=1,NP
   35 XPXIN(I,J)=XPXIN(I,J)+DTRAN(I,K)*DTRAN(J,K)
      XPXIN(J,I)=XPXIN(I,J)
   37 CONTINUE
   38 CONTINUE
C
C*****COMPUTE INVERSE OF THE SQUARE DESIGN MATRIX.
      NPP=NP
      NFI=NF
      CALL INVERS(XPXIN,NFI)
C  ---IF MATRIX SINGULAR, PRINT DIAGNOSTIC AND STOP.
      IF(NFI.LT.0) GO TO 44
C
C*****PRINT DESIGN MATRIX TRANSPOSE, AND INVERSE MATRIX.
      LINE=LINE+3
      CALL PAGE
      WRITE(6,221)
      DO 40 I=1,NF
      LINE=LINE+(NP-1)/10+1
      CALL PAGE
      WRITE(6,225) (DTRAN(I,J),J=1,NP)
   40 CONTINUE
      LINE=LINE+3
```

```
      CALL PAGE
      WRITE(6,222)
      DO 41 I=1,NF
      LINE=LINE+(NF-1)/10+1
      CALL PAGE
      WRITE(6,225) (XPXIN(I,J),J=1,NF)
   41 CONTINUE
      RETURN
   44 WRITE (6,224)
      LINE=LINE+1
      CALL PAGE
      RETURN
  220 FORMAT(44H THE SYSTEM MATRICES WITH DESIGN CENTER AT (,
     1  F8.2,1H,,F8.2, 6H) ARE,)
  221 FORMAT(/17H SYSTEM TRANSPOSE/)
  222 FORMAT(/12H X'X INVERSE/)
  224 FORMAT(54H X'X MATRIX IS SINGULAR, FURTHER CALCULATIONS DELETED.)
  225 FORMAT(1X,10E13.5)
      END
```

```
      SUBROUTINE INVERS (A,N)
C
C           THIS SUBROUTINE INVERTS MATRIX A AND PUTS THE RESULT BACK
C     INTO A.  COMPUTATION IS DONE IN DOUBLE PRECISION USING MATRIX S.
C        N = INPUT TO THE ROUTINE AS THE ORDER OF THE MATRIX A.
C        --------IS MADE = -1 IF THE MATRIX IS SINGULAR.
      DIMENSION A(9,9),S(9,18)
      DOUBLE PRECISION  S,BUFF,DVH,FPY
      K=N+N
      DO 12 I=1,N
    3 DO 5 J=1,K
      S(I,J)=0.D0
    5 CONTINUE
      DO 10 J=1,N
   10 S(I,J)=DBLE(A(I,J))
      IN=I+N
      S(I,IN)=1.D0
   12 CONTINUE
C          COMPUTE INVERSE
   15 DO 150 I=1,N
      L=I
      M=I+1
      JIN=I
   20 IF (S(I,I).NE.0.0) GO TO 45
   25 LE=I+1
   26 IF (S(LE,I).NE.0.0) GO TO 800
   27 LE=LE+1
   28 IF (LE-N) 26,26,900
  800 DO 35 J=1,K
      BUFF=S(I,J)
      S(I,J)=S(LE,J)
      S(LE,J)=BUFF
   35 CONTINUE
   41 GO TO 20
   45 DVH=S(I,I)
      DO 46 J=1,K
   46 S(I,J)=S(I,J)/DVH
      S(I,I)=1.0D0
   48 IF (I.GE.N) GO TO 149
   49 FPY=S(M,L)
      IF (FPY.EQ.0.D0) GO TO 75
   50 DO 70 J=1,K
      BUFF=FPY*S(I,J)
      S(M,J)=S(M,J)-BUFF
   70 CONTINUE
   75 JIN=M+1
      IF (JIN.GT.N) GO TO 149
  100 M=M+1
  120 GO TO 49
  149 CONTINUE
  150 CONTINUE
      DO 385 I=2,N
```

```
      L=I
      M=I-1
  350 FPY=S(M,L)
      IF (FPY.EQ.0.00) GO TO 375
  351 DO 370 J=1,K
      BUFF=FPY*S(I,J)
      S(M,J)=S(M,J)-BUFF
  370 CONTINUE
  375 IF (M.LE.1) GO TO 384
  380 M=M-1
      GO TO 350
  384 CONTINUE
  385 CONTINUE
  390 DO 402 I1=1,N
      LL=I1
  395 DO 400 J1=1,N
      KK=N+J1
  396 A(I1,J1)=SNGL(S(LL,KK)+S(LL,KK))-SNGL(S(LL,KK))
  400 CONTINUE
  402 CONTINUE
      RETURN
C         NO INVERSE
  900 N=-1
      RETURN
      END
```

```
      SUBROUTINE DEFINE
C
C      CONVERTS THE LIMITS OF THE DESIGN REGION FROM POLAR TO RECTANGULAR
C     COORDINATES.
C         XN = MINIMUM X (INPUT AS A RADIUS, OUTPUT IN CARTESIAN).
C         XP = MAXIMUM X
C         YN = MINIMUM Y (INPUT AS AN ANGLE, CONVERTED TO CARTESIAN).
C         YP = MAXIMUM Y.
      COMMON/MTOVR/XN,XP,YN,YP
      A=XN
      B=XP
      C=YN*0.017453293
      D=YP*0.017453293
      IF((YN.GT.0.).AND.(YP.GT.0.)) GO TO 17
      IF((YN.LT.0.).AND.(YP.LT.0.)) GO TO 12
      YN=B*SIN(C)
      YP=B*SIN(D)
      XN=A*COS(C)
      IF(ABS(YN).LT.ABS(YP)) XN=A*COS(D)
      RETURN
   12 YN=B*SIN(C)
      YP=A*SIN(D)
      XN=A*COS(C)
      XP=B*COS(D)
      RETURN
   17 YN=A*SIN(C)
      YP=B*SIN(D)
      XN=A*COS(D)
      XP=B*COS(C)
      RETURN
      END
```

```
      SUBROUTINE CONVRT(NP)
C
C     CONVERTS X,Y IN POLAR COORDINATES TO RECTANGULAR COORDINATES.
      COMMON/MATOD/X(15),Y(15),DUM(150)
      DO 5 I=1,NP
      R=Y(I)*0.017453293
      A=X(I)
      X(I)=A*COS(R)
      Y(I)=A*SIN(R)
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REGRES(NP,SIGZXY,SIGZ,A)
C COMPUTES FACTORS OF REGRESSION.
C
C*****INPUT ARGUMENTS************************
C          A(I) = COEFFICIENTS OF BASIS FUNCTIONS.
C          D(I,J) = INVERSE OF THE SQUARE DESIGN MATRIX.
C          NF = NO. OF BASIS FUNCTIONS USED.
C          NP = NO. OF POINTS IN THE DESIGN REGION.
C          Z(I) = RESPONSE VALUE AT THE POINTS WITHIN THE DESIGN REGION.
C*****FACTORS OF REGRESSION COMPUTED.**********************
C          FRGR = REGRESSION/RESIDUE OF PLANE.
C          RFIT = RESIDUE OF FIT = (Z-ZCAL)**2.
C          RGRSN = REGRESSION = RESIDUE OF (PLANE-FIT).
C          RPLANE = RESIDUE OF PLANE = (Z-ZBAR)**2.
C          SIGZ = VARIANCE OF Z.
C          SIGZXY = VARIANCE OF Z.XY.
C          ZBAR = AVERAGE Z WITHIN THE DESIGN REGION.
C          ZCAL = PREDICTED Z (FROM REGRESSION).
C****************************************************
      COMMON/ALL/NF,B(9),DI(9,9),XC,YC,DUMM(3)
      COMMON/MATOD/DUM(30),D(9,15),Z(15)
      COMMON/LINENO/LINE,PAGENO
      DIMENSION A(9),FBASIS(9)
      DATA FBASIS/1H ,1HX,1HY,2HXY,7H3X**2-2,7H3Y**2-2,
     1 9H3YX**2-2Y,9H3XY**2-2X,8HB(5)B(6)/
C
C*****INITIALIZE
      RNP=NP
      RFIT=0.0
      ZBAR=0.0
      ZSQS=0.0
C
C*****PERFORM CALCULATIONS
      DO 11 I=1,NP
      ZBAR=ZBAR+Z(I)
      ZSQS=ZSQS+Z(I)**2
      ZCAL=0.0
      DO 10 K=1,NF
   10 ZCAL=ZCAL+A(K)*D(K,I)
   11 RFIT=RFIT+(Z(I)-ZCAL)**2
      IF((NP-NF).EQ.0) GO TO 12
      SIGZXY=RFIT/(NP-NF)
      GO TO 13
   12 SIGZXY=-0.
   13 RPLANE=ZSQS-ZBAR**2/RNP
      SIGZ=RPLANE/(NP-1)
      RGRSN=RPLANE-RFIT
      IF(RPLANE.EQ.0.) GO TO 14
      FRGR=RGRSN/RPLANE
      GO TO 15
   14 FRGR=0.
   15 ZBAR = ZBAR/RNP
C
C*****PRINT RESULTS.
      LINE=LINE+12
      CALL PAGE
      WRITE(6,1) XC,YC,RPLANE,RFIT,RGRSN,FRGR
      IF(SIGZXY.EQ.-0.) GO TO 16
      WRITE(6,2) SIGZXY,SIGZ, ZBAR
      GO TO 17
   16 WRITE(6,4) SIGZ,ZBAR
   17 WRITE(6,3) (A(K),FBASIS(K),K=1,NF)
    1 FORMAT(1H0,//,* REGRESSION ABOUT THE POINT (*,F7.2,*,*,F7.2,*)*/
     1* RESIDUE OF PLANE = *,E17.9/* RESIDUE OF FIT = *,E17.9/* REGRESSI
     2ON = *,E17.9,/* REGRESSION/RESIDUE OF PLANE = *,E17.9)
    2 FORMAT(*  SIGMA SQUARED OF Z.XY = *,E17.9,*   SIGMA SQUARED OF Z =
     1 *,E17.9,*     ZBAR = *, E12.6)
    3 FORMAT(*0ZHAT = *,E11.5,1X,A1,*  +  *,2(E11.5,2X,A1,*  +  *),E11.5
     1,2X,A2,*  +  *,E11.5,*  (*,A7,*)  +  *,/,8X,E11.5,*  (*,A7,
     2 3(*)  +  *,E11.5,*  (*,A8))
    4 FORMAT(*  SIGMA SQUARED OF Z.XY =  (NO ESTIMATE)      SIGMA SQUARED
     1 OF Z = *,E17.9,*     ZBAR = *,E12.6)
      RETURN
      END
```

```
      SUBROUTINE PAGE
C
C     PRINTS TITLE AND PAGE NUMBER AT THE TOP OF EACH OUTPUT PAGE.
      INTEGER PAGENO
      DIMENSION TITLE(5)
      COMMON/LINENO/LINE,PAGENO,TITLE
C
C     IF THE BOTTOM OF THE PRESENT PAGE HAS BEEN REACHED, PRINT THE TITLE
C         ON THE NEXT PAGE.
      IF (LINE.GT.56) 5,10
    5 LINE=0
      PAGENO=PAGENO+1
      WRITE(6,7) TITLE,PAGENO
    7 FORMAT(*1*,/50X,5A10,5X,*PAGE *,I5/)
   10 RETURN
      END
```

```
      SUBROUTINE PRINPLT(Z,NI,NJ,SF,DXT,DYT,XMINT,YMAXT,XINC,YINC)
C     THIS SUBROUTINE PRINTS A CONTOUR AREA SUMMARY, PLOTS A 3D RESPONSE
C          SURFACE, AND PLOTS A CONTOUR MAP FOR BOTH THE VARIABLE BEING
C         EVALUATED (Z) AND ITS VARIANCE (VAR).  IT WILL ALSO PLOT 2D XZ
C         OR YZ PLOTS OF THE VARIABLE RESPONSE IF REQUESTED.
C
C
C*****ARGUMENT LIST*****************
C       DXT,DYT = THE DISTANCE BETWEEN THE Z POINTS IN THE X AND Y DIRECTIONS.
C       LABEL = THE LABEL USED FOR PLOTS.
C        NCV = THE NO. OF CONTOUR LEVELS OF THE RESPONSE VARIANCE PLOT (MAX=15)
C       NCZ = THE NO. OF CONTOUR LEVELS ON THE RESPONSE PLOT (MAX=15).
C            ---IF POSITIVE CONTOUR LEVELS ARE CALCULATED USING ZMIN,
C                    ZMAX, AND NCZ.
C            --- IF NEGATIVE, CONTOUR LEVELS ARE READ IN.
C       NJ, NI  = THE DIMENSIONS OF Z IN THE X AND Y DIRECTIONS RESPECTIVELY.
C       NIX, NIY = THE NO. OF SAMPLE POINTS BETWEEN LINES PLOTTED ON THE
C        3D RESPONSE SURFACE IN THE X AND Y DIRECTIONS.
C       NPLOT = THE NO. OF OPTIONAL 2D XZ AND YZ PLOTS OF THE RESPONSE
C          SURFACE (MAX=10).
C       NVAR = FLAG FOR VARIANCE RESPONSE SURFACE AND CONTOUR PLOTS.
C            ----IF = 0, VARIANCE PLOTS PROVIDED.
C            ----IF = 1, VARIANCE PLOTS NOT PROVIDED.
C       PLOT(I) = --IF POSITIVE, THE X VALUE OF A REQUESTED YZ 2D PLOT.
C                 --IF NEGATIVE, THE Y VALUE OF A REQUESTED XZ 2D PLOT.
C       SF = THE AREA ON THE XY PLANE REPRESENTED BY ONE Z POINT.
C       TITLE = THE HOLERITH IDENTIFIER OF Z
C        XINC, YINC = INCRIMENT (IN X AND Y DIRECTIONS) BETWEEN SUCCESS-
C              IVE DESIGN SYSTEMS.
C       XMINT, YMINT = THE MINIMUM X AND Y VALUES ( THE COORDINATES OF Z(1,1))
C       XUNITS = HOLERITH IDENTIFIER OF THE X AND Y UNITS.
C       Z(I,J) = THE VARIABLE RESPONSE AT EVENLY SPACED POINTS ON THE XY PLANE.
C       ZMIN, ZMAX = CONVENIENT MINIMUM AND MAXIMUM Z VALUES (ALSO THE
C          SMALLEST AND LARGEST CONTOUR LEVEL).
C       ZPTS(I) = AN ARRAY CONTAINING THE TOTAL AREA BOUNDED BY EACH
C          CONTOUR LEVEL
C       ZUNITS = HOLERITH IDENTIFIER OF THE RESPONSE VALUE UNITS.
C***************************************
C
      DIMENSION Z(91,91),ZPTS(25),TITLE(5),LABEL(8),PLOT(10),CLVL(25)
      COMMON VAR(91,91)
      COMMON/LINENO/LINE,PAGENO,TITLE
      LINE = 60
      CALL PAGE
C
C*****READ DATA
      READ(5,101) ZMIN,ZMAX,NCZ,NCV,NPLOT,NIX,NIY,NVAR
      IF(NCZ.GT.0) GO TO 3
      I =-NCZ
      READ(5,102) (CLVL(J),J=1,I)
    3 IF(NPLOT.EQ.0) GO TO 1
```

```
      READ(5,102) (PLOT(I),I=1,NPLOT)
    1 NSTOP=0
C
C*****INITIAL CALCULATIONS
      FNJ=NJ
      FNI = NI
      NSJ=((FNJ-1.)*DXT)/(.99999*XINC)
      NSI=((FNI-1.)*DYT)/(.99999*YINC)
      XXX=(FNJ-1.)*DXT + XMINT
      YYY=YMAXT - (FNI-1.)*DYT
      ZINC=0.
      TOTAL=FLOAT(NI*NJ)*SF
    2 DO 4 I=1,25
    4 ZPTS(I)=0.
C
C*****IF CONTOUR VALUES NOT READ IN, CALCULATE.
      IF(NCZ.LT.0) GO TO 6
      ZINC=(ZMAX-ZMIN)/FLOAT(NCZ-1)
      CV=ZMIN-ZINC
      DO 5 I=1,NCZ
      CV=CV+ZINC
    5 CLVL(I)=CV
      GO TO 7
    6 NCZ=-NCZ
C
C*****CALCULATE THE NO. OF POINTS ABOVE EACH CONTOUR LEVEL (ZPTS(I)'.
    7 DO 42 I=1,NI
      DO 42 J=1,NJ
      DO 41 K=1,NCZ
      IF(Z(I,J).LT.CLVL(K)) GO TO 42
   41 ZPTS(K)= ZPTS(K) + 1.
   42 CONTINUE
C
C*****CALCULATE THE AREA ABOVE EACH CONTOUR LEVEL (ZPTS(I)).
      DO 45 I=1,NCZ
   45 ZPTS(I)=ZPTS(I)*SF
C
C*****READ LABELS AND UNITS
      READ(5,103) TITLE,ZUNITS,XUNITS
C
C*****WRITE CONTOUR SUMMARY TABLES.
      WRITE(6,110) TITLE,ZUNITS,XUNITS
      DO 50 I=1,NCZ
      PERCNT=(ZPTS(I)/TOTAL)*100.
   50 WRITE(6,111)CLVL(I),ZPTS(I),PERCNT
C
C*****PLOT RESPONSE SURFACE
      CALL OPTION(0,1,0,0,2)
      CALL PWRT(50,36,TITLE,50,2,0)
      CALL OPTION(0,0,0,0,0)
      CALL RSPSUR(Z,NI,NJ,DXT,DYT,ZMIN,ZMAX,NIY,NIX)
C
```

```
C*****PLOT CONTOUR MAP
      CALL OPTION(0,1,0,0,2)
      CALL PWRT(130,100,TITLE,50,2,0)
      CALL OPTION(0,0,0,0,0)
      ENCODE(28,115,LABEL) ZUNITS
      CALL PWRT(130,74,LABEL,28,0,0)
      ENCODE(64,118,LABEL) XMINT,XXX,XUNITS
      CALL PWRT(130,58,LABEL,64,0,0)
      ENCODE(64,119,LABEL) YYY,YMAXT,XUNITS
      CALL PWRT(130,42,LABEL,64,0,0)
      ENCODE(66,116,LABEL)DXT,XUNITS
      CALL PWRT(130,26,LABEL,66,0,0)
      ENCODE(66,117,LABEL)DYT,XUNITS
      CALL PWRT(130,10,LABEL,66,0,0)
      CALL CALCNT(Z,NI,NJ,NCZ,CLVL,DYT,DXT)
      MMM=(NJ-1)/NSJ
      NNN=(NI-1)/NSI
      CALL PERIM(NSJ,MMM,NSI,NNN)
      CALL FRAME
C
C*****IF BOTH RESPONSE AND VARIANCE PLOTTED, STOP.
      IF(NSTOP.EQ.1) RETURN
C
C*****MAKE XZ AND YZ PLOTS
      IF(NPLOT.EQ.0) GO TO 55
      LBLX=7HX VALUE
      LBLY= 7HY VALUE
      CALL GRDFMT(7H(F10.3),7H(F10.3))
      DO 55 I=1,NPLOT
      IF(PLOT(I).LT.0) GO TO 52
C
C     YZ PLOTS
      JJ=(PLOT(I)-XMINT)/DXT + 1.
      ZZZ=ZMIN- (ZMAX-ZMIN)/FLOAT(NCZ-1)
      CALL SET(.1,.95,.15,1.,YYY,YMAXT,ZZZ ,ZMAX,1)
      CALL PERIML(NSI,1,NCZ,1)
      ENCODE(64,112,LABEL) TITLE,ZUNITS
      CALL PWRT(6,200 ,LABEL,64,1,1  )
      ENCODE(42,113,LABEL) PLOT(I)
      CALL PWRT(500,60,LBLY,7,1,0)
      CALL PWRT(100,12,LABEL,42,1,0)
      CALL FRSTPT(YMAXT,Z(1,JJ))
      DO 51 II=2,NI
      FI=II-1
      X=YMAXT - FI*DYT
   51 CALL VECTOR(X,Z(II,JJ))
      CALL FRAME
      GO TO 55
C
C     XZ PLOTS
   52 PLOT(I) =-PLOT(I)
      II=(YMAXT -PLOT(I))/DYT + 1.
      ZZZ=ZMIN - (ZMAX-ZMIN)/FLOAT(NCZ-1)
      CALL SET(.1,.95,.15,1.,XMINT,XXX,ZZZ ,ZMAX,1)
      CALL PERIML(NSJ,1,NCZ,1)
      ENCODE(64,112,LABEL) TITLE,ZUNITS
      CALL PWRT(6,200 ,LABEL,64,1,1)
      ENCODE(42,114,LABEL) PLOT(I)
      CALL PWRT(100,12,LABEL,42,1,0)
      CALL PWRT(500,60,LBLX,7,1,0)
      CALL FRSTPT(XMINT,Z(II,1))
      N=NJ-1
      DO 53 J=1,N
      FJ=J
      X=XMINT + FJ*DXT
   53 CALL VECTOR(X,Z(II,J))
      CALL FRAME
   55 CONTINUE
C
C*****IF VARIANCE PLOTS NOT REQUESTED, STOP.
      IF(NVAR.EQ.1) RETURN
C
C*****REPLACE Z, ZMIN, ZMAX, ZINC, AND NCZ WITH VARIANCE PARAMETERS.
      NSTOP = 1
      ZMIN=VAR(1,1)
      ZMAX=VAR(1,1)
      DO 60 I=1,NI
      DO 60 J=1,NJ
      IF(VAR(I,J).LT.ZMIN) ZMIN=VAR(I,J)
      IF(VAR(I,J).GT.ZMAX) ZMAX=VAR(I,J)
   60 Z(I,J)=VAR(I,J)
      NCZ=NCV
C
C*****DO CONTOUR SUMMARY, RESPONSE PLOT,  AND CONTOUR PLOT FOR VARIANCE.
      GO TO 2
  101 FORMAT(2F10.0,6I5)
  102 FORMAT(10F8.0)
  103 FORMAT(7A10)
  110 FORMAT(//*0CONTOUR SUMMARY OF *,5A10,//4X,*CONTOUR LEVEL*,5X,*AREA
     1 COVERED*,6X,*PERCENT OF*,/,4X,*(*,A10,*)*,6X,*(SQ.*,A8,*)*5X*TOTA
     2L AREA*)
  111 FORMAT(1H ,4X,E11.5,5X,E13.7,7X,F8.3)
  112 FORMAT(5A10,*IN  *,A10)
  113 FORMAT(*GRAPH OF RESPONSE SURFACE AT X=*,F10.3)
  114 FORMAT(*GRAPH OF RESPONSE SURFACE AT Y=*,F10.3)
  115 FORMAT(*CONTOUR VALUES IN *,A10)
  116 FORMAT(*EACH TICK MARK REPRESENTS *,F10.5,2X,A10,*IN THE X DIRECTI
     1ON*)
  117 FORMAT(26X,F10.5,2X,A10,*IN THE Y DIRECTION*)
  118 FORMAT(*THE MAP BOUNDARIES ARE X= *,F10.3,* TO X= *,F10.3,1X,A10)
  119 FORMAT(19X,*AND Y= *,F10.3,* TO Y= *,F10.3,1X,A10)
      END
```

```
      SUBROUTINE RSPSUR(Z,I,J,XD,YD,ZMIN,ZMAX,N,M)
      DIMENSION Z(91,91)
C       THIS SUBROUTINE PLOTS A RESPONSE SURFACE IN THREE DIMENSIONS USING
C       THE MICROFILM PLOTTER.  LINES ARE PLOTTED ON EQUADISTANT X-Z, AND
C       Y-Z  PLANES TO MAKE UP THE SURFACE.
C
C******ARGUMENT LIST********
C         Z = AN I-BY-J ARRAY CONTAINING THE Z-VALUES (RESPONSE) AT EACH
C              EVENLY SPACED PLOTTING POINT ON THE X-Y PLANE.  IT IS
C              ASSUMED THAT Z(1,1) IS LOCATED AT COORDINATES (XMIN,YMAX).
C         I = THE NUMBER OF PLOTTING POINTS IN THE Y-DIRECTION.
C         J = THE NUMBER OF PLOTTING POINTS IN THE X-DIRECTION
C         XD = THE DISTANCE BETWEEN PLOTTING POINTS IN THE X-DIRECTION
C         YD = THE DISTANCE BETWEEN PLOTTING POINTS IN THE Y-DIRECTION.
C         ZMIN= MINIMUM Z-VALUE.
C         ZMAX = MAXIMUM Z-VALUE.
C            -----ZMIN AND ZMAX SHOULD BE CONVENIENT NUMBERS WHICH BOUND
C               THE DATA.
C         N = THE NUMBER OF SAMPLE POINTS BETWEEN LINES ON THE PLOTTED
C             SURFACE IN THE Y DIRECTION.
C         M = THE NUMBER OF SAMPLE POINTS BETWEEN LINES IN THE X-DIRECTION.
C*****************************************
C
C       SCALE THE DRAWING.
      A=J
      B=I
      YC=.707*YD/XD
      ZC=.5*YC*B/(ZMAX-ZMIN)
      YMIN=ZC*ZMIN
      YMAX =YC*B + ZC*ZMAX
      XMAX=A+YC*B
      YMX=1.5*YC*B
      IF(XMAX.GE.YMX)  GO TO 10
      XL=(YMX -XMAX)/(2.*YMX)  + .05
      XR=1.-(YMX -XMAX)/(2.*YMX)
      CALL SET(XL,XR,.05,1.,0.,XMAX,YMIN,YMAX,1)
      GO TO 20
   10 YL=(XMAX-YMX) /(2.*XMAX) + .05
      YH=1.-(XMAX-YMX) /(2.*XMAX)
      CALL SET(.05,1.,YL,YH,0.,XMAX,YMIN,YMAX,1)
   20 CONTINUE
C
C       DRAW THE BASELINES
      X=YC + 1.
      Y=ZMIN*ZC + YC
      CALL FRSTPT(X,Y)
      X=YC + A
      CALL VECTOR(X,Y)
      Y=ZMIN*ZC + YC*B
      CALL VECTOR(XMAX,Y)
C
C       DRAW THE HORIZONTAL LINES
      DO 31 II=1,I,N
      BB=II
      IB = I-II+1
      Y1=BB*YC
      Y=Z(IB,1)*ZC+Y1
      X=Y1 + 1.
      CALL FRSTPT(X,Y)
      DO 30 JJ=2,J
      AA=JJ
      Y=Z(IB,JJ)*ZC+Y1
      X=Y1+AA
   30 CALL VECTOR(X,Y)
      Y=Y1 + ZMIN*ZC
   31 CALL VECTOR(X,Y)
C
C       DRAW THE VERTICLE LINES
      DO 40 JJ=1,J,M
      AA=JJ
      X=YC + AA
      Y=ZMIN*ZC + YC
      CALL FRSTPT(X,Y)
      Y=Z(I,JJ) *ZC + YC
      CALL VECTOR(X,Y)
      DO 40 II=2,I
      IB=I-II+1
      BB=II
      X=BB*YC + AA
      Y=Z(IB,JJ)*ZC+BB*YC
   40 CALL VECTOR (X,Y)
      CALL FRAME
      RETURN
      END
```

```
      SUBROUTINE CALCNT(AM,MX,NY,NC,CLVL,DX,DY)
C     THIS SUBROUTINE MAKES A CONTOUR PLOT OF DATA CONTAINED IN ARRAY AM.
C     IT LABELS THE CONTOURS AND PRINTS AN H OR L AT EACH LOCAL HIGH OR
C     LOW ON THE PLOT.
C
C     IN THIS VERSION OF CALCNT, POINT (1,1) OF ARRAY AM IS IN THE UPPER
C     LEFT HAND CORNER OF THE PLOT, AND POINT (MX,1) IS IN THE LOWER LEFT
C     HAND CORNER OF THE PLOT.
C
C*****INPUT PARAMETERS**********************************************************
C           AM(I,J) = ARRAY TO BE CONTOURED.
C           MX = NO. OF POINTS IN THE Y DIRECTION (IN AM) TO BE PLOTTED
C                (FIRST SUBSCRIPT).
C           MY = NO. OF POINTS IN THE X DIRECTION TO BE PLOTTED (SECOND
C                SUBSCRIPT).
C           NC = NO. OF CONTOUR LEVELS TO BE PLOTTED.  (MAX=25)
C           CLVL(I) = CONTOUR LEVELS TO BE PLOTTED.   (MAX=25)
C           DX = SPACING IN THE Y DIRECTION OF SAMPLE POINTS IN AM.
C           DY = SPACING IN THE X DIRECTION OF SAMPLE POINTS IN AM.
C*****************************************************
C
      COMMON/CONT/MT,NT,IX,IY,IDX,IDY,ISS,NP,CV,NNT,ASH,INX(8),INY(8),
     1IPT(3,3),LEGEND(11),REC(500) ,NQ,SBL,CSBL,LBLF,XC,YC
      DIMENSION AM(91,91),SYBL(25),CLVL(25)
      DATA SYBL/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1HA,1HB,1HC,1HD,1HE,
     11HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,1HO,1HP/
C   INITIALIZE
   81 NQ=1
C
      MT=MX
      NT=NY
      NNT=2
      IDASH=1777B
      IPT(1,1)=8
      IPT(1,2)=1                                                        SCAN0320
      IPT(1,3)=2                                                        SCAN0330
      IPT(2,1)=7                                                        SCAN0340
      IPT(2,3)=3                                                        SCAN0350
      IPT(3,1)=6                                                        SCAN0360
      IPT(3,2)=5                                                        SCAN0370
      IPT(3,3)=4                                                        SCAN0380
      INX(1)=-1                                                         SCAN0390
      INX(2)=-1                                                         SCAN0400
      INX(3)=0                                                          SCAN0410
      INX(4)=1                                                          SCAN0420
      INX(5)=1                                                          SCAN0430
      INX(6)=1                                                          SCAN0440
      INX(7)=0                                                          SCAN0450
      INX(8)=-1                                                         SCAN0460
      INY(1)=0                                                          SCAN0470
      INY(2)=1                                                          SCAN0480
      INY(3)=+1                                                         SCAN0490
      INY(4)=+1                                                         SCAN0500
      INY(5)=0                                                          SCAN0510
      INY(6)=-1                                                         SCAN0520
      INY(7)=-1                                                         SCAN0530
      INY(8)=-1                                                         SCAN0540
      FM=MT
      FN=NT
      X4=FM*DX
      Y4=FN*DY
C      THE SCALING HAS BEEN CHANGED TO REORIENT PLOT(NEXT 10 STATEMENTS)
      IF(X4-Y4)21,21,22
   21 Y2=.985
      X2 = (X4/Y4)*.85+.13
      GO TO 23
   22 X2=.98
      Y2 = (Y4/X4)*.85+.125
   23 CALL SET(.125,Y2,.13,X2,1.,FN,-FM,-1.,1)
      XC=(FM-1.)/((X2-.125)*1024.)
      YC=(FN-1.)/((Y2-.13)*1024.)
   24 CONTINUE
      ENCODE(25,202,LEGEND)
  202 FORMAT(*   CONTOUR$CSYMBOL VALUES*)
      CALL PWRT(4,908,LEGEND,25,0,0)
C   DETERMINE CURRENT LEVEL TO BE CONTOURED
C
      DO 150 I=1,NC
      SBL=SYBL(I)
      CV=CLVL(I)
      ENCODE(13,203,LEGEND) SBL,CV
  203 FORMAT(1A1,*=*,E11.5)
      MMM=890 - I*16
      CALL PWRT(4,MMM,LEGEND,13,0,0)
C
      CALL SCAN(AM,MT,NT)
C
  150 CONTINUE
C
   32 CALL HILO (AM,MT,NT)
      RETURN
      END                                                               DCNT0510
```

```
      SUBROUTINE SCAN(AM,M,N)
C
C     THIS SUBROUTINE SCANS AM FOR THE STARTING POINTS OF CONTOURS.
C
      COMMON/CONT/MT,NT,IX,IY,IDX,IDY,ISS,NP,CV,NNT,ASH,INX(8),INY(8),
     1IPT(3,3),LEGEND(11),REC(500) ,NQ,SBL,CSBL,LBLF,XC,YC
      DIMENSION AM(91,91)
      NP=0                                                              SCAN0230
      DO 58 J=1,500
   58 REC(J)=0                                                          SCAN0580
      ISS=0                                                             SCAN0590
    2 MT1=MT-1                                                          SCAN0600
      DO 110 I=1,MT1                                                    SCAN0610
      IF (AM(I,1)-CV) 55,110,110                                        SCAN0620
   55 IF (AM(I+1,1)-CV) 110,57,57                                       SCAN0630
   57 IX=I+1                                                            SCAN0640
      IY=1                                                              SCAN0650
      IDX=-1                                                            SCAN0660
      IDY=0                                                             SCAN0670
      LBLF=1
      CSBL =  1. - YC*6.
      CALL LINEAR (AM, MT,NT)
  110 CONTINUE                                                          SCAN0690
      NT1=NT-1                                                          SCAN0700
      DO 20 I=1,NT1                                                     SCAN0710
      IF (AM(MT,I)-CV) 15,20,20                                         SCAN0720
   15 IF (AM(MT,I+1)-CV)  20,17,17                                      SCAN0730
   17 IX=MT                                                             SCAN0740
      IY=I+1                                                            SCAN0750
      IDX=0                                                             SCAN0760
      IDY=-1                                                            SCAN0770
      LBLF=-1
      Q=MT
      CSBL =-(Q + XC*10.)
      CALL LINEAR (AM, MT,NT)
   20 CONTINUE                                                          SCAN0790
   22 DO 30 I=1,MT1                                                     SCAN0800
      MT2=MT+1-I                                                        SCAN0810
      IF (AM(MT2,NT)-CV) 25,30,30                                       SCAN0820
   25 IF (AM(MT2-1,NT)-CV)  30,27,27                                    SCAN0830
   27 IX=MT2-1                                                          SCAN0840
      IY=NT                                                             SCAN0850
      IDX=1                                                             SCAN0860
      IDY=0                                                             SCAN0870
      LBLF=1
      Q=NT
      CSBL =  Q + YC*6.
      CALL LINEAR (AM, MT,NT)
   30 CONTINUE                                                          SCAN0890
      DO 40 I=1,NT1                                                     SCAN0900
      NT2=NT+1-I                                                        SCAN0910
      IF (AM(1,NT2)-CV)  35,40,40                                       SCAN0920
   35 IF (AM(1,NT2-1)-CV)   40,37,37                                    SCAN0930
   37 IX=1                                                              SCAN0940
      IY=NT2-1                                                          SCAN0950
      IDX=0                                                             SCAN0960
      IDY=1                                                             SCAN0970
      LBLF=-1
      CSBL=-(1. - XC*10.)
      CALL LINEAR (AM, MT,NT)
   40 CONTINUE                                                          SCAN0990
      ISS=1                                                             SCAN1000
      NT1=NT-1                                                          SCAN1010
      MT1=MT-1                                                          SCAN1020
      LBLF=0
      DO 10 J=2,NT1                                                     SCAN1030
      DO 10 I=1,MT1                                                     SCAN1040
      IF (AM(I,J)-CV)  5,10,10                                          SCAN1050
    5 IF (AM(I+1,J)-CV)  10,7,7                                         SCAN1060
    7 COM   =100*(I+1)+J
      IF (NP) 12,11,12                                                  SCAN1080
   12 DO 9 ID= 1,NP                                                     SCAN1090
      IF (REC(ID)-COM    ) 9,10,9
    9 CONTINUE                                                          SCAN1110
   11 IX= I+1                                                           SCAN1120
      IY=J                                                              SCAN1130
      IDX=-1                                                            SCAN1140
      IDY=0                                                             SCAN1150
      CALL LINEAR (AM, MT,NT)
   10 CONTINUE                                                          SCAN1170
      RETURN                                                            SCAN1180
      END                                                               SCAN1190
```

```
      SUBROUTINE LINEAR(AM,IDIM,JDIM)
C
C     THIS SUBROUTINE PLOTS THE CONTOURS.
C
      COMMON/CONT/MT,NT,IX,IY,IDX,IDY,ISS,NP,CV,NNT,ASH,INX(8),INY(8),
     1IPT(3,3),LEGEND(11),REC(500) ,NQ,SBL,CSBL,LBLF,XC,YC
      DIMENSION AM(91,91)
      N=1
      IXO=IX                                                            TRAC0190
      IYO=IY                                                            TRAC0200
      ISX=IDX +2                                                        TRAC0210
      ISY=IDY+2                                                         TRAC0220
      IS=IPT(ISX,ISY)                                                   TRAC0230
      ISO=IS                                                            TRAC0250
      IF(ISO-8)1,1,17
   17 ISO=ISO-8                                                         TRAC0270
    1 IF (IDX) 10,2,10                                                  CALC0150
    2 X=IX
      Z=IY                                                              CALC0170
      IY2=IY+IDY                                                        CALC0180
      DY=IDY                                                            CALC0190
      Y     = ((AM(IX,IY)-CV)/(AM(IX,IY)-AM(IX,IY2))) *DY + Z           C
      GO TO 54
   10 Y=IY
      W=IX                                                              CALC0230
      DX=IDX                                                            CALC0240
      IX2=IX+IDX                                                        CALC0250
      X     = ((AM(IX,IY)-CV)/(AM(IX,IY)-AM(IX2,IY))) *DX +W
   54 IF (IS.EQ.1)    306,49
  306 NP=NP+1                                                           TRAC0810
      IF(NP.GT.500)  WRITE(6,7777)
 7777 FORMAT(1H ,*NEED MORE REC*)
      REC(NP)=100*IX+IY                                                 TRAC0820
   49 IS=IS+1                                                           TRAC0390
    9 IF (IS-9)   8, 7,7                                                TRAC0410
    7 IS=IS-8                                                           TRAC0420
    8 IDX=INX(IS)                                                       TRAC0430
      IDY=INY(IS)                                                       TRAC0440
      IX2=IX+IDX                                                        TRAC0450
      IY2=IY+IDY                                                        TRAC0460
      IF(N) 67,73
   67 IF(LBLF) 120,122,121
  120 CALL PWRT(Y,CSBL,SBL,1,0,0)
      GO TO 123
  121 CALL PWRT(CSBL,-X,SBL,1,0,0)
      GO TO 123
  122 IF(N.NE.1) GO TO 124
       R1=0.
       OLDX=X
      OLDY=Y
      N=2
      GO TO 51
  124 R=SQRT(((X-OLDX)/XC)**2 + ((Y-OLDY)/YC)**2)
      R1=R1 + R
      IF(R1.LT.8.)  GO TO 125
      R1=R - (R1-8.)
      XN=((X-OLDX)*R1)/R + OLDX
      YN=((Y-OLDY)*R1)/R + OLDY
      IF(N.EQ.2) GO TO 126
C       REORIENTED PLOT
      CALL FRSTPT(YN,-XN)
      CALL VECTOR(Y,-X)
      N=0
      GO TO 51
  126 R1=0.
C       REORIENTED PLOT
      CALL PWRT(YN,-XN,SBL,1,0,0)
      N=3
      OLDX=XN
      OLDY=YN
      GO TO 124
  125 OLDX=X
      OLDY=Y
      GO TO 51
C     REORIENTED PLOT
  123 CALL FRSTPT(Y,-X)
      N=0
      GO TO 51
C     REORIENTED PLOT
   73 CALL VECTOR(Y,-X)
   51 IF(ISS)20,58
   20 IF(IX-IXO)  12,21,12                                              TRAC0550
   21 IF(IY-IYO)  12,22,12                                              TRAC0560
   22 IF(IS-ISO)  12,23,12                                              TRAC0570
   23 IF (IS.EQ.1) 307,14
  307 NP=NP+1                                                           TRAC0890
      IF(NP.GT.500)  WRITE(6,7777)
      REC(NP)=100*IX+IY                                                 TRAC0900
   14 IF (IDX) 52,53,52
   53 X=IX
      Z=IY                                                              CALC0170
      IY2=IY+IDY                                                        CALC0180
      DY=IDY                                                            CALC0190
      Y     = ((AM(IX,IY)-CV)/(AM(IX,IY)-AM(IX,IY2))) *DY + Z
C     REORIENTED PLOT
      CALL VECTOR(Y,-X)
   74 RETURN
   52 Y=IY
      W=IX                                                              CALC0230
      DX=IDX                                                            CALC0240
      IX2=IX+IDX                                                        CALC0250
      X     = ((AM(IX,IY)-CV)/(AM(IX,IY)-AM(IX2,IY))) *DX +W
C     REORIENTED PLOT
```

```
      CALL VECTOR(Y,-X)
      RETURN
   58 IF(IX2) 13,75,13
   13 IF (IX2-MT) 19,19,76
   19 IF(IY2) 11,77,11
   11 IF (IY2-NT)  12,12,78
   12 IF (CV-AM(IX2,IY2))  16,16,55
   55 ISTE=(IS/2)*2
      IF (ISTE.EQ.IS) 49,1
   16 IS=IS+5                                                          TRAC0830
      IX=IX2                                                           TRAC0840
      IY=IY2                                                           TRAC0850
      GO TO 9                                                          TRAC0860
C     REORIENTED PLOT (NEXT 13 STATEMENTS)
   75 CSBL=-(1. - XC*10.)
      CALL PWRT(Y,CSBL,SBL,1,0,0)
      RETURN
   76 Q=MT
      CSBL =-(Q + XC*10.)
      CALL PWRT(Y,CSBL,SBL,1,0,0)
      RETURN
   77 CSBL =  1. - YC*6.
      CALL PWRT(CSBL,-X,SBL,1,0,0)
      RETURN
   78 Q=NT
      CSBL =  Q + YC*6.
      CALL PWRT(CSBL,-X,SBL,1,0,0)
      RETURN
      END                                                              TRAC0960
```

```
      SUBROUTINE HILO(AM,M,N)
C
C     THIS SUBROUTINE PRINTS AN H AT EACH LOCAL HIGH POINT AND AN L AT
C          EACH LOCAL LOW POINT.
C
      COMMON/CONT/MT,NT,IX,IY,IDX,IDY,ISS,NP,CV,NNT,ASH,INX(8),INY(8),
     1IPT(3,3),LEGEND(11),REC(500) ,NQ,SBL,CSBL,LBLF,XC,YC
      DIMENSION AM(91,91), JSIGN(2)
      DATA(JSIGN=1HL,1HH)
  100 FORMAT (E10.3)
      NMT=NNT+1
      NN=NT-NNT
      MM=MT-NNT
      DO 10 J=NMT,NN
      DO 10 I=NMT,MM
      II=I-NNT
   11 IF (AM(I,J)-AM(II,J))12,10,13
   12 KS=1
      DO 40 K=1,NNT
      DO 40 KK=1,7
      IX2=I+K*INX(KK)
      IY2=J+K*INY(KK)
      IF(AM(I,J)-AM(IX2,IY2))40,10,10
   40 CONTINUE
      GO TO 30
   13 KS=2
      DO 50 K=1,NNT
      DO 50 KK=1,7
      IX2=I+K*INX(KK)
      IY2=J+K*INY(KK)
      IF(AM(I,J)-AM(IX2,IY2))10,10,50
   50 CONTINUE
   30 XPLT = I
      YPLT = J
C     REORIENTED PLOT
      CALL PSYM(YPLT,-XPLT,JSIGN(KS),0,0,1)
   10 CONTINUE
      RETURN
      END
```

```
      PROGRAM ECOGRID(INPUT,PUNCH ,TAPE5=INPUT,TAPE6=PUNCH)
C     THIS PROGRAM PUNCHES THE GRID FOR PROGRAM ECOMAP
C*****INPUT VARIABLES
C         YL,YH = MINIMUM AND MAXIMUM Y-COORDINATES FOR THE GRID
C         XL,XH = MINIMUM AND MAXIMUM X-COORDINATES FOR THE GRID
C          XINC = INCRIMENT BETWEEN GRID POINTS IN THE X DIRECTION
C          YINC = INCRIMENT BETWEEN GRID POINTS IN THE Y DIRECTION
      READ(5,100)XL,XH,YL,YH,XINC,YINC
      NUMX=(XH-XL+XINC)/XINC
      NUMY=(YH-YL+YINC)/YINC
      X=XL-XINC
      DO 10 I=1,NUMX
      X=X+XINC
      Y=YH+YINC
      DO 10 J=1,NUMY
      Y=Y-YINC
      IF(X.EQ.XL.OR.X.EQ.XH)GO TO 9
      IF(Y.EQ.YL.OR.Y.EQ.YH)GO TO 9
      X1=X-XINC
      X2=X+XINC
      Y1=Y-YINC
      Y2=Y+YINC
      WRITE(6,200)X,Y,X1,X2,Y1,Y2
      GO TO 10
    9 WRITE(6,200)X,Y
   10 CONTINUE
  100 FORMAT(6F10.0)
  200 FORMAT(6F8.2)
      END
```

Example

THE SYSTEM MATRICES WITH DESIGN CENTER AT (400.00, 1200.00) ARE,

SYSTEM TRANSPOSE

.10000E+01	.10000E+01	.10000E+01	.10000E+01	.10000E+01	.10000E+01	.10000E+01	.10000E+01	.10000E+01
-.10000E+01	-.10000E+01	-.10000E+01	0.	0.	0.	.10000E+01	.10000E+01	.10000E+01
.10000E+01	0.	-.10000E+01	.10000E+01	0.	-.10000E+01	.10000E+01	0.	-.10000E+01
-.10000E+01	0.	.10000E+01	0.	0.	0.	.10000E+01	0.	-.10000E+01
.10000E+01	.10000E+01	.10000E+01	-.20000E+01	-.20000E+01	-.20000E+01	.10000E+01	.10000E+01	.10000E+01
.10000E+01	-.20000E+01	.10000E+01	.10000E+01	-.20000E+01	.10000E+01	.10000E+01	-.20000E+01	.10000E+01
.10000E+01	0.	-.10000E+01	-.20000E+01	0.	.20000E+01	.10000E+01	0.	-.10000E+01
-.10000E+01	.20000E+01	-.10000E+01	0.	0.	0.	.10000E+01	-.20000E+01	.10000E+01

X≠X INVERSE

.11111E+00	0.	0.	0.	0.	0.	0.	0.
0.	.16667E+00	0.	0.	0.	0.	0.	0.
0.	0.	.16667E+00	0.	0.	0.	0.	0.
0.	0.	0.	.25000E+00	0.	0.	0.	0.
0.	0.	0.	0.	.55556E-01	0.	0.	0.
0.	0.	0.	0.	0.	.55556E-01	0.	0.
0.	0.	0.	0.	0.	0.	.83333E-01	0.
0.	0.	0.	0.	0.	0.	0.	.83333E-01

REGRESSION ABOUT THE POINT (400.00,1200.00)
RESIDUE OF PLANE = .170682222E+03
RESIDUE OF FIT = .382336111E+02
REGRESSION = .132448611E+03
REGRESSION/RESIDUE OF PLANE = .775995352E+00
SIGMA SQUARED OF Z.XY = .382336111E+02 SIGMA SQUARED OF Z = .213352778E+02 ZBAR = .514444E+01

ZHAT = .51444E+01 + -.17333E+01 X + -.12333E+01 Y + .12500E+00 XY + .32222E+00 (3X**2-2) +
-.81111E+00 (3Y**2-2) + .12583E+01 (3YX**2-2) + .24583E+01 (3XY**2-2) +

REGRESSION ABOUT THE POINT (400.00, 800.00)
RESIDUE OF PLANE = .189762222E+03
RESIDUE OF FIT = .621469444E+02
REGRESSION = .127615278E+03
REGRESSION/RESIDUE OF PLANE = .672500966E+00
SIGMA SQUARED OF Z.XY = .621469444E+02 SIGMA SQUARED OF Z = .237202778E+02 ZBAR = .715556E+01

ZHAT = .71556E+01 + -.27167E+01 X + -.11833E+01 Y + -.22750E+01 XY + .66111E+00 (3X**2-2) +
.79444E+00 (3Y**2-2) + .40833E+00 (3YX**2-2) + -.16583E+01 (3XY**2-2) +